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Tensor decomposition exploiting diversity of propagation velocities: Application to localization of icequake events

Francesca Raimondi [a,*], Pierre Comon [a], Olivier Michel [a], Souleymen Sahnoun [a], Agnes Helmstetter [b]

[a] CNRS, University Grenoble Alpes, Gipsa-Lab, F-38000 Grenoble, France
[b] ISTerre, CNRS, OSUG-C, 38400 Saint Martin d'Heres Cedex, France

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ABSTRACT

The problem of direction of arrival (DoA) estimation of seismic plane waves impinging on an array of sensors is considered from a new deterministic perspective using tensor decomposition techniques. In addition to temporal and spatial sampling, further information is taken into account, based on the different propagation speed of body waves (P and S) through solid media. Performances are evaluated through simulated data in terms of the Cramér–Rao bounds and compared to other reference methods such as ESPRIT and MUSIC, in the presence of additive Gaussian circular noise. The proposed approach is then applied to real seismic data recorded at the Argentiére glacier, occurring at the interface between the ice mass and the underlying bedrock. MUSIC and ESPRIT rely on the estimation of the covariance matrix of received data, thus requiring a large number of time samples. Moreover, information about propagation speed diversity is not taken into account by existing models in array processing. The discovered advantage in terms of the average error in estimating the direction of arrival of body waves is noteworthy, especially for a low number of sensors, and in separating closely located sources. Additionally, an improvement of precision in processing real seismic data is observed.

1. Introduction

In many cases, the superimposition principle applies in practical problems, provided the nonlinearity domain is not reached (turbulence and saturation). This allows us to model physical phenomena as linear combinations of a few simpler ones. In this paper, we are interested in the decomposition of a multivariate function into a sum of functions whose variables separate. In particular, this simplified model is relevant in narrow-band antenna array processing in the far-field, which we consider in the present framework.

In the context of seismic monitoring, seismology aims at studying waves generated by rupture phenomena taking place within a volume of interest (rock and ice). Although the most interesting events take place at a certain depth – mostly unknown – within the analyzed volume, acquisition systems and sensor arrays are most often located close to the surface. The main quantity to be measured is ground displacement (in the form of its derivative – velocity – or its second derivative – acceleration), produced by impinging elastic waves. The localization of the sources forms the first requirement of data analysis, in order to prevent damage provoked by seismic events, and to monitor the activity of complex structures such as glaciers or volcanoes. Seismic arrays, after being introduced in the 1960s, have made essential contributions to this problem. These arrays consist of numerous seismometers placed at discrete points in space in a well-defined configuration [1]: apart from an improvement of SNR by combining individual sensor recordings, they have

* Corresponding author.
E-mail address: francesca.raimondi@gipsa-lab.grenoble-inp.fr (F. Raimondi).

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been used to refine models of the Earth's interior, through classical tools such as beamforming, slant stacking techniques and frequency-wave number analysis.

In wider terms, direction of arrival (DoA) estimation is a central problem in array signal processing, concerning several areas of engineering including telecommunications, speech, astronomy, seismology, and medical applications. Array processing requires a set of multiple sensors placed at different positions in space, receiving source signals from different directions [2,3]. Among techniques aiming at estimating directions of arrival, some need to resort to an exhaustive search, like beamforming and MUSIC (MUltiple SIgnal Classification) [4], whereas others do not, like root-MUSIC [5] and ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) [6]. Whenever propagation speed is considered as a constant v, traditional array processing only relies on a temporal and spatial sampling of the propagating wavefield. MUSIC algorithm is based on the spectral decomposition of the sample covariance matrix under the spatially white noise assumption (to be presented in Appendix B). This method has the advantage of being asymptotically *statistically efficient*, unlike beamforming techniques, despite a serious sensitivity to SNR and resolution limitations for correlated or closely spaced sources [3]. Moreover, the algorithm requires the perfect knowledge of the position of each sensor. ESPRIT applies to an array composed of two identical subarrays displaced one from the other according to an unknown translation vector, whereas the calibration of the array is needed otherwise. The concept of signal-subspace processing embodied by MUSIC and ESPRIT, originally derived for narrow band and stationary signals, can be generalized to the wide-band case [7–9].

A deterministic approach based on tensor decomposition has been introduced in [10], through the extension of the rotational invariance principle to more than one displacement. It provides the localization of more sources than sensors in each subarray, with less restrictive requirements for signal stationarity than the afore-mentioned statistical methods. The advantage of tensor decompositions lies in the need for shorter data records, since the estimation of statistical quantities from available samples is not a requirement anymore. Furthermore, like ESPRIT, it allows to estimate the impinging signals up to a scale factor, without resorting to a spatial matched filter. The tensor model puts forward parsimony and separability [11]:

1. Parsimony expresses a function g as a finite sum of simpler constituents

$$g = \sum_{r=1}^{R} \varsigma_r \, h_r \tag{1}$$

2. Separability decouples a function h that depends on multiple factors into a product of simpler constituents $\phi_d, d = 1, \ldots, D$, each one depending only on one factor $\boldsymbol{x}_d$

$$h(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_D) = \prod_{d=1}^{D} \phi_d(\boldsymbol{x}_d)$$

In the field of array processing for source separation and DoA estimation, R refers to the number of sources impinging on an array, and D to the tensor order, i.e. the dimension of multilinearity within the model

$$g(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_D) = \sum_{r=1}^{R} \varsigma_r \prod_{d=1}^{D} \phi_{rd}(\boldsymbol{x}_d)$$

Tensor decomposition derives from the need to solve the *inverse problem*, i.e., the identification of factors ϕ_{rd} based on noisy measurements of g: as it will be hereafter discussed, the direction of arrival can be extracted after the resolution of this problem. For this purpose, the measurements are stored in a multidimensional array and decomposed into a sum of *rank one* terms [12,10]. A *decomposable* three-way tensor can be defined by a vector triplet[1]

$$d_{lmk} = a_l b_m c_k \text{ or equivalently } \mathcal{D} = \boldsymbol{a} \otimes \boldsymbol{b} \otimes \boldsymbol{c}$$

Any order-3 tensor admits a decomposition into a sum of decomposable tensors

$$\mathcal{M} = \sum_{r=1}^{R} \varsigma_r \mathcal{D}(r) \tag{2}$$

where coefficients ς_r can always be chosen to be real positive, and decomposable tensors $\mathcal{D}(r)$ to have unit norm, *i.e.* for Euclidean norm, $\|\mathcal{D}\| = \|\boldsymbol{a}\| \, \|\boldsymbol{b}\| \, \|\boldsymbol{c}\| = 1$. The minimal value of R such that this decomposition holds is called the tensor rank: if R is not too large, the corresponding decomposition is unique and deserves to be referred to as Canonical Polyadic (CP); other terminologies include *rank decomposition* or *Candecomp/Parafac*. Hence, decomposable tensors have a rank equal to 1, by definition [12]. Now in terms of coordinates, tensor $\mathcal{M}$ is represented by a $L \times M \times K$ three-way array, which consequently decomposes as

$$M_{lmk} = \sum_{r=1}^{R} \varsigma_r A_{lr} B_{mr} C_{kr} \tag{3}$$

where the three factor matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ have unit norm columns. This is equivalent to the general R-term trilinear model

$$\mathcal{M} = \sum_{r=1}^{R} \varsigma_r \boldsymbol{a}_r \otimes \boldsymbol{b}_r \otimes \boldsymbol{c}_r \tag{4}$$

where each array $\boldsymbol{a}_r \otimes \boldsymbol{b}_r \otimes \boldsymbol{c}_r$ is a rank-1 array.

This paper is aimed at exploiting *another type of diversity*, in addition to spatial and temporal sampling traditionally employed in array processing (cf. Section 3 for a detailed explanation of the concept of diversity): the propagation speed diversity[2] of body waves through solids, namely pressure (P) and shear (S) waves. Current array processing methods like [4,6] only focus on information conveyed by a single body wave, like the P wave, whereas the contents delivered by the other is somehow wasted. Our approach intends to exploit this information as a whole, whereas translational invariance used in [10] is no longer necessary.

This paper is organized as follows. Section 2 presents the physical model and the assumptions. Section 3.1 makes a synthesis of the main narrowband 2-D preexisting algorithms. Our deterministic method, exploiting the propagation speed

[1] Once bases in every linear space are fixed, tensors are defined by their array of coordinates. See [12] for details.

[2] Since the focus is on narrow-band processing, the distinction between group and phase propagation velocities is irrelevant.