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Pair formation models for sexually transmitted infections: A primer

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ABSTRACT

For modelling sexually transmitted infections, duration of partnerships can strongly influence the transmission dynamics of the infection. If partnerships are monogamous, pairs of susceptible individuals are protected from becoming infected, while pairs of infected individuals delay onward transmission of the infection as long as they persist. In addition, for curable infections re-infection from an infected partner may occur. Furthermore, interventions based on contact tracing rely on the possibility of identifying and treating partners of infected individuals. To reflect these features in a mathematical model, pair formation models were introduced to mathematical epidemiology in the 1980's. They have since been developed into a widely used tool in modelling sexually transmitted infections and the impact of interventions. Here we give a basic introduction to the concepts of pair formation models for a susceptible-infected-susceptible (SIS) epidemic. We review some results and applications of pair formation models mainly in the context of chlamydia infection.

1. Introduction

In classical epidemic models, essential underlying assumptions are that contacts are instantaneous and every contact is with another individual of the population (Diekmann, Heesterbeek, & Britton, 2012). For many infectious diseases these assumptions are reasonable and lead to good results. For example, for modelling the spread of influenza or measles, taking instantaneous contacts into account has resulted in valid descriptions of transmission dynamics and these models have been used successfully to assess the impact of vaccination (Anderson & May 1991). For modelling dynamics of sexually transmitted infections (STI) the situation is different, as these assumptions may not always be valid. If individuals of a population form long lasting partnerships and have repeated contacts with the same individual over a long time period, this influences the transmission risk of an infection that spreads via those contacts. If partnerships are monogamous, individuals in a pair of two susceptibles are protected from becoming infected as long as the partnership lasts. A partnership has to dissolve and a new one be formed before transmission to another person can occur. Also, if a person who is in a partnership with an infectious partner recovers, he or she may acquire a re-infection from his/her infected partner. Such effects are especially influential for infections with relatively long infectious periods and for populations where individuals have few but long lasting

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partnerships. If duration of infection and duration of partnerships are of the same order of magnitude, partnership dynamics interacts with disease transmission and determines the potential of the disease to establish itself in a population.

To deal with such dynamic properties of sexually transmitted infections, pair formation models were first introduced into the field of infectious disease modelling by Dietz and Hadeler (Dietz & Hadeler, 1988). They modified models from mathematical demography to include transmission of infection in an age-structured two-sex population. Simplified versions of this pair formation epidemic model were later formulated by Kretzschmar and Dietz (Kretzschmar & Dietz, 1998) and compared with models, which do not take partnership duration into account. Since then, pair formation models have been used in many variants, have been implemented into simulation models for STI (Kretzschmar, van Duynhoven, & Severijnen, 1996; Low et al., 2007; Turner et al., 2006), and have been applied to analyse the impact of various types of interventions (Heijne, Althaus, Herzog, Kretzschmar, & Low, 2011; Powers et al., 2011). Pair formation models in structured populations have been analysed mathematically by Hadeler et al. (Hadeler, 2012; Hadeler, Waldstätter, & Wörz-Busekros, 1988). More recently, extensions to the pair formation approach which are capable of describing also concurrent partnerships have been derived and analysed (Leung, Kretzschmar, & Diekmann, 2012, 2015).

Here we review the approach of pair formation models with the aim of giving an easy introduction into the main ideas and related literature for readers, who want to obtain a quick overview. We introduce and explain assumptions and structure of simple pair formation models. We formulate models for one-sex populations for reasons of simplicity, but all models can easily be extended to two-sex populations. We give some examples for the application of pair formation models, where we focus mainly on curable sexually transmitted infections (STI) with *chlamydia trachomatis* (chlamydia) as an example.

2. Modelling partnership dynamics

We start with introducing a model for the partnership dynamics without infection. We assume that the population is subdivided into singles (denoted by $X(t)$) and pairs of individuals (denoted by $P(t)$). The total population size is given by $N = X + 2P$, because a pair consists of 2 individuals. Singles form new pairs with a rate ρ and pairs separate with rate σ. New individuals enter the population with a constant recruitment rate B and leave the population (by death or ceasing sexual activity) with rate μ. We further assume that $X(t) \geq 0$ and $P(t) \geq 0$ for all t. The pair formation process is then described by the system of differential equations

$$\frac{dX}{dt} = B - \mu X - \rho X + 2\sigma P + 2\mu P$$

$$\frac{dP}{dt} = \frac{1}{2}\rho X - \sigma P - 2\mu P$$

This system of equations has a unique equilibrium given by

$$X^\star = \frac{B(\sigma + 2\mu)}{\mu(\rho + \sigma + 2\mu)}$$

$$P^\star = \frac{B\rho}{2\mu(\rho + \sigma + 2\mu)}$$

In equilibrium we have $N = B/\mu$. Therefore, a fraction $x^\star = \frac{\sigma+2\mu}{\rho+\sigma+2\mu}$ is single. The fraction of people in a partnership is given by $2p^\star = 2P^\star/N = \frac{\rho}{(\rho+\sigma+2\mu)}$. In the following we will always assume that the pair formation and separation process is at equilibrium, i.e. that there is a constant proportion of singles and paired individuals in the population.

The fractions of singles and paired individuals can be used to estimate ρ and σ from sexual behaviour data. For example, if we observe that survey participants report 1.5 new partners per year and 70% of participant report that they are in a partnership, we can set

$$\rho x^\star = 1.5$$

$$2p^\star = 0.7$$

If we assume for simplicity that $B = \mu = 0$, we get $\rho = 5$ and $\sigma = 2.1$. Note that these estimates were derived under the assumption that there are no concurrent partnerships.

We can also use the equations to derive some other relevant quantities describing the typical life course of an individual. The fraction of the population that is in a partnership

$$2p^\star = \frac{\rho}{\rho + \sigma + 2\mu}$$