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Applying Empirical Mode Decomposition and mutual information to separate stochastic and deterministic influences embedded in signals

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ABSTRACT

Empirical Mode Decomposition (EMD) is a method to decompose signals into Intrinsic Mode Functions (IMFs) to be analyzed in terms of instantaneous frequencies and amplitudes. By comparing the phase spectra of IMFs, we observed that a subset of them contains more stochastic influences while the other is predominantly deterministic. Considering this observation, we claim that IMFs can be combined to form two additive components: one deterministic and another stochastic. Having both components separated, researchers can improve data modeling as well as forecasting. In this context, this paper presents a new approach to separate deterministic from stochastic influences embedded in signals, considering the mutual information contained in phase spectra of consecutive IMFs. As previous step of this study, we also proved that EMD works as a filter bank.

1. Introduction

The Empirical Mode Decomposition (EMD) method was designed to decompose signals into components referred to as Intrinsic Mode Functions (IMFs) in order to study and analyze their instantaneous frequencies and amplitudes by using the Hilbert Spectral Analysis (HSA) [1]. Among the main advantages of EMD are the extraction of a reduced number of components in comparison to other techniques and its application to linear and nonlinear signals [1].

By analyzing the EMD results, we empirically observed that this method produces IMFs at different frequency bandwidths. This same observation also caught the attention of other researchers such as Flandrin et al. [2], who studied EMD using numerical experiments. From this empirical observation, in this paper we prove that EMD works as a filter bank. While conducting this proof and experimenting with EMD on synthetic and real-world datasets we also confirmed that EMD separates deterministic from stochastic influences embedded in signals.

While experimenting of real-world data we noticed high-frequency IMFs could be better modeled using Stochastic tools [3], while low-frequency ones could be better modeled using Dynamical System tools [4]. In fact, frequency by itself cannot be used as an indicator of stochasticity or determinism; however, we observed that the way EMD produces IMFs could help to separate deterministic from stochastic components. In order to proceed with this separation, we designed a new approach that compares the phase spectra (complex Fourier coefficients) between consecutive IMFs to measure their similarities. As a consequence, similar IMFs share some degree of information what allows us to assume them as deterministic while dissimilar ones are taken as stochastic.

This phase spectra comparison was performed using the Mutual Information method (MI), which measures the dependency between variables. Experiments performed

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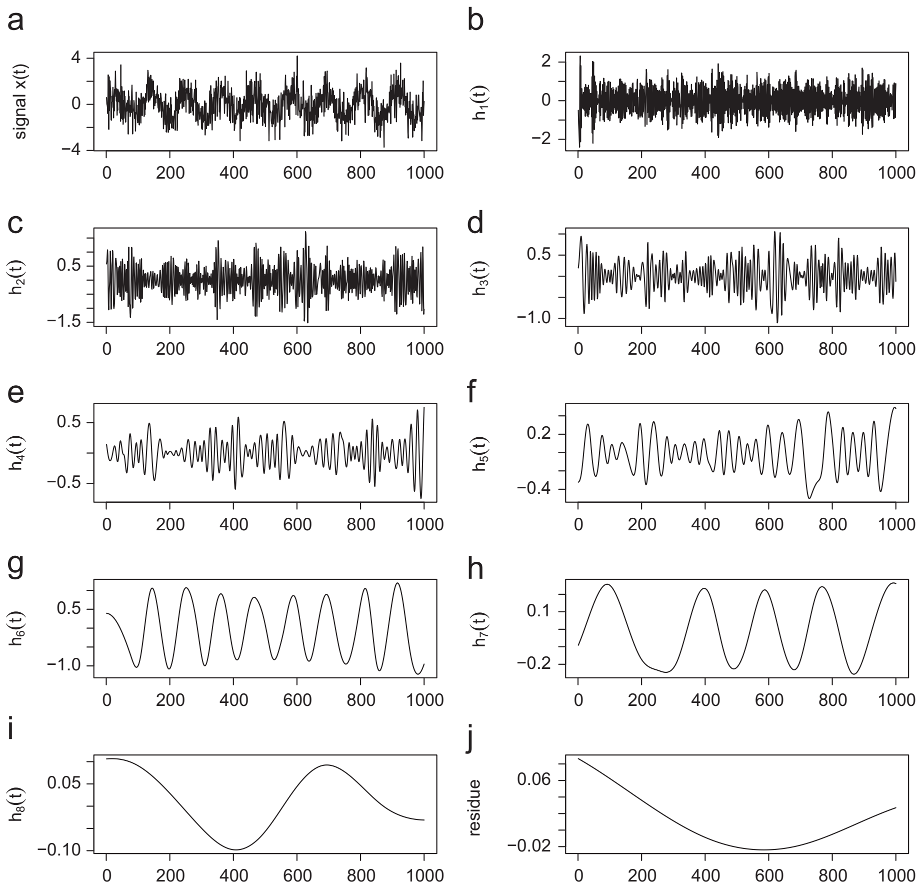

Fig. 1. The noisy signal $x(t) = \sin(2\pi t) + \varepsilon(0, 1)$ is shown in plot (a). Plots from (b) to (i) show all IMFs $h_n(t)$ extracted at each iteration from the signal $x(t)$ using the EMD method. The last plot (j) shows the residue $r(t)$.

on signals with additive noise confirmed that the first high-frequency IMFs produced by the EMD method present lower levels of mutual information, i.e., they usually share almost no information; however, as next IMFs were produced (at lower frequencies), higher mutual information was observed. In this sense, when two IMFs are stochastic, no relevant information is shared between them and, consequently, their mutual information is low. On the other hand, higher levels of mutual information point out the presence of similar influences which we assume as determinism.

Considering this analysis, we state that the application of EMD on signals provides IMFs that can be separated into two classes: one with lower mutual information and another presenting higher mutual information levels. The first class corresponds to stochastic influences, whereas the second represents deterministic ones. Having the deterministic and the stochastic components separated, one can improve: (i) the modeling and prediction of each individual component [5] as well as (ii) the filtering, once noise is typically related to stochastic processes [6,7].

The remainder of this paper is organized as follows. In Section 2, we present an overview on the Empirical Mode Decomposition (EMD) method. The process of extracting components from signals, using EMD, was analyzed in Section 3 considering instantaneous frequencies and phase spectra. This analysis has motivated the development of this work, which starts discussing the Nyquist–Shannon sampling theorem in Section 4. This theorem was considered to prove EMD works as a filter bank as detailed in Section 5. Next (Section 6), we present the experimental results performed on synthetic and real-world datasets to illustrate our proof and confirm that IMFs can be combined to form two components: one stochastic and another deterministic. Finally, in Section 7, we draw conclusions and discuss future work.

2. On empirical mode decomposition

The Empirical Mode Decomposition (EMD) method supports the decomposition of signals into Intrinsic Mode Functions (IMFs) regardless of their linearity, stationarity, and stochasticity [1,2,8]. The key point to perform this decomposition is the sifting process [9,10], which initially analyzes a signal $x(t)$ and identifies local maxima and minima values of observations along time. The cubic spline method is then applied to the maxima and minima to compose the upper $u(t)$ and lower $l(t)$ envelopes, respectively [1]. The approximation values obtained using both cubic splines (upper and lower) are used to compute the mean envelope $m(t)$.