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Noise benefits in joint detection and estimation problems ☆

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ABSTRACT

Adding noise to inputs of some suboptimal detectors or estimators can improve their performance under certain conditions. In the literature, noise benefits have been studied for detection and estimation systems separately. In this study, noise benefits are investigated for joint detection and estimation systems. The analysis is performed under the Neyman–Pearson (NP) and Bayesian detection frameworks and according to the Bayesian estimation criterion. The maximization of the system performance is formulated as an optimization problem. The optimal additive noise is shown to have a specific form, which is derived under both NP and Bayesian detection frameworks. In addition, the proposed optimization problem is approximated as a linear programming (LP) problem, and conditions under which the performance of the system can or cannot be improved via additive noise are obtained. With an illustrative numerical example, performance comparison between the noise enhanced system and the original system is presented to support the theoretical analysis.

1. Introduction

Although an increase in the noise power is generally associated with performance degradation, addition of noise to a system may introduce performance improvements under certain arrangements and conditions in a number of electrical engineering applications including neural signal processing, biomedical signal processing, lasers, nano-electronics, digital audio and image processing, analog-to-digital converters, control theory, statistical signal processing, and information theory, as exemplified in [1] and references therein. In the field of statistical signal processing, noise benefits are investigated in various studies such as [2–17]. In [2], it is shown that the detection probability of the optimal detector for a described network with nonlinear elements driven by a weak sinusoidal signal in white Gaussian noise is non-monotonic with respect to the noise power and fixed false alarm probability; hence, detection probability enhancements can be achieved via increasing the noise level in certain scenarios. For an optimal Bayesian estimator, in a given nonlinear setting, with examples of a quantizer [3] and phase noise on a periodic wave [4], a non-monotonic behavior in the estimation mean-square error is demonstrated as the intrinsic noise level increases. In [5], the proposed simple suboptimal nonlinear detector scheme, in which the detector parameters are chosen according to the system noise level and distribution, outperforms the matched filter under non-Gaussian noise in the Neyman–Pearson (NP) framework. In [6], it is noted that the performance of some optimal detection strategies display a non-monotonic behavior with respect to the noise root-mean square amplitude in a binary hypothesis testing problem with a nonlinear setting, where non-Gaussian noise (two different distributions are examined for numerical purposes: Gaussian mixture and uniform distributions) acts on the

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phase of a periodic signal. In [16] and [17], theoretical conditions are provided related to improvability and non-improvability of suboptimal detectors for weak signal detection via noise benefits.

One approach for realizing noise benefits is to tune the parameters of a nonlinear system, as employed, e.g., in [8–12]. An alternative approach is the injection of a random process independent of both the meaningful information signal (transmitted or hidden signal) and the background noise (undesired signal). It is firstly shown by Kay in [13] that addition of independent randomness may improve suboptimal detectors under certain conditions. Later, it is proved that a suboptimal detector in the Bayesian framework may be improved (i.e., the Bayes risk can be reduced) by adding a constant signal to the observation signal; that is, the optimal probability density function is a single Dirac delta function [14]. This intuition is extended in various directions and it is demonstrated that injection of additive noise to the observation signal at the input of a suboptimal detector can enhance the system performance [15,18–34]. In this paper, performance improvements through noise benefits are addressed in the context of *joint detection and estimation* systems by adding an independent noise component to the observation signal at the input of a suboptimal system. Notice that the most critical keyword in this approach is suboptimality. Under non-Gaussian background noise, optimal detectors/estimators are often nonlinear, difficult to implement, and complex systems [35,36]. Hence, the main aim is to improve the performance of a fairly simple and practical system by adding specific randomness (noise) at the input.

Chen et al. revealed that the detection probability of a suboptimal detector in the NP framework can be increased via additive independent noise [15]. They examined the convex structure of the problem and specified the nature of the optimal probability distribution of additive noise as a probability mass function with at most two point masses. This result is generalized for *M*-ary composite hypothesis testing problems under NP, restricted NP and restricted Bayes criteria [25,29,34]. In estimation problems, additive noise can also be utilized to improve the performance of a given suboptimal estimator [4,19,30]. As an example of noise benefits for an estimation system, it is shown that Bayesian estimator performance can be enhanced by adding non-Gaussian noise to the system, and this result is extended to the general parameter estimation problem in [19]. As an alternative example of noise enhancement application, injection of noise to blind multiple error rate estimators in wireless relay networks is presented in [30].

In this study, noise benefits are investigated for a joint detection and estimation system, which is presented in [37]. Without introducing any modification to the structure of the system, the aim is to improve the performance of the joint detection and estimation system by only adding noise to the observation signal at the input. Therefore, the detector and the estimator are assumed to be given and fixed. In [37], optimal detectors and estimators are derived for this joint system. However, the optimal structures may be overcomplicated for an implementation. In this study, it is assumed that the given joint detection and estimation system is suboptimal, and the purpose is defined as the examination of the performance improvements via additive noise under this assumption. The main contributions of this study can be summarized as follows:

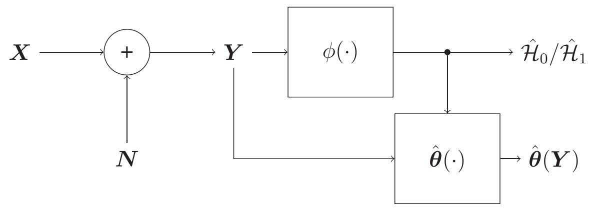

Fig. 1. Joint detection and estimation scheme with noise enhancement: The only modification on the original system is the introduction of the additive noise **N**.

- Noise benefits are investigated for joint detection and estimation systems for the first time.
- Both Bayesian and NP detection frameworks are considered, and the probability distribution of the optimal additive noise is shown to correspond to a discrete probability mass function with a certain number of point masses under each framework.
- For practical applications in which additive noise can take finitely many different values, a linear programming (LP) problem is formulated to obtain the optimal additive noise.
- Necessary and sufficient conditions are derived to specify the scenarios in which additive noise can or cannot improve system performance.

In addition, theoretical results are also illustrated on a numerical example and noise benefits are investigated from various perspectives.

2. Problem formulation

Consider a joint detection and estimation system as illustrated in Fig. 1, where the aim is to investigate possible improvements on the performance of the system by adding "noise" **N** to observation **X**. In other words, instead of employing the original observation **X**, the system operates based on the noise modified observation **Y**, which is generated as follows:

$$\mathbf{Y} = \mathbf{X} + \mathbf{N}. \tag{1}$$

The problem is defined as the determination of the optimum probability distribution for the additive noise without modifying the given joint detection and estimation system; that is, detector $\phi(\cdot)$ and estimator $\hat{\theta}(\cdot)$ are fixed. Also, the additive noise **N** is independent of the observation signal **X**.

For the joint detection and estimation system, the model in [37] is adopted. Namely, the system consists of a detector and an estimator subsequent to it, and the detection is based on the following binary composite hypothesis testing problem [37]:

$$\begin{aligned} \mathcal{H}_0&: \quad \mathbf{X} \sim f_0^X(\mathbf{x}) \\ \mathcal{H}_1&: \quad \mathbf{X} \sim f_1^X(\mathbf{x}|\boldsymbol{\Theta} = \boldsymbol{\theta}), \quad \boldsymbol{\Theta} \sim \pi(\boldsymbol{\theta}) \end{aligned} \tag{2}$$