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Robust closed-form time-of-arrival source localization based on α-trimmed mean and Hodges–Lehmann estimator under NLOS environments

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ABSTRACT

In this paper, we propose an NLOS source localization method that utilizes the robust statistics, namely, the α-trimmed mean and Hodges–Lehmann estimator. The root mean squared error average of the proposed methods is similar to that of the other estimators such as M-estimator and Taylor-series maximum likelihood estimator using the median, but the proposed robust estimators have advantages that they have the closed-form solution. The simulation results show that the root mean squared error performance of the proposed methods is similar or outperforms that of the iteration-based M-estimator. The Taylor-series maximum likelihood estimator based on the sample median is most superior among the investigated localization methods, but it has the disadvantages that the computational complexity is high and that the solution may converge to the local maxima. Also, it is shown that the performances of the closed-form proposed estimators outperform the JMAP-ML and LS estimator in the above of certain NLOS noise level.

1. Introduction

Source localization is a technique that finds a geometrical point of intersection using the measurements from each receiver, such as the time difference of arrival (TDOA), the time of arrival (TOA), or the received signal strength (RSS). Localizing point sources, using passive and stationary sensors, is of considerable interest and this has been a repeated theme of research in the radar, sonar, global positioning system, video conferencing, and telecommunication areas. Even though location estimation problems have been investigated extensively in the existing literature, there are still some unresolved problems. One of the key challenges in localization problem is to estimate the position of the source in dense cluttered non-line-of-sight (NLOS) environments. NLOS scenarios occur when there is an obstruction between transmitter and receiver which are encountered in indoor environments and outdoor situations such as urban area. Several methods exist for multi-sensor localization [1–6], but none of these works address the methods to mitigate the NLOS problem. In general, the research fields of localization for NLOS problem can be categorized into three parts, i.e., (1) constrained least squares (LS) method using the optimization such as the linear programming, quadratic programming, expectation maximization (EM), and joint maximum a posteriori-maximum likelihood (JMAP-ML) methods [7–10], (2) localization based on the "NLOS identify and discard" [11–15], (3) localization using the robust statistics [16–23]. The localization using the optimization method has the comparatively high accuracy, however it has a disadvantage that the computational load is higher than that of the analytical solution. The localization using the "NLOS identify and

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discard" method has also the high accuracy when the NLOS sensors are perfectly separated from the LOS sensors. However, the complete classification of LOS sensors and NLOS sensors is nearly impossible, thus the classification error evidently exists and this false classification incurs the localization error drastically increased. Furthermore, when the number of sensors is large, the number of cases to be calculated increases, thus the computational burden is much high. To overcome these disadvantages of the above existing robust localization methods, we propose the closed-form NLOS source localization methods based on the robust statistics. The approaches to estimate the center of distribution, such as the mean or median, using the robust statistics have been existed in the statistical society, e.g., M-estimator, least median squares (LMS), L-estimator, R-estimator, etc. [16,17]. The example of the L-estimator is the α-trimmed mean and that of the R-estimator is the Hodges–Lehmann estimator. It is known that these two estimators have high robustness when outliers exist and efficiency when the outlier does not exist. Furthermore, they have the lower computational complexity compared to the above counterparts and can be represented as the closed-form solution. However, these two methods have not been adopted in the source localization context for NLOS environments despite these advantages. Thus, in this paper, we propose the α-trimmed mean and Hodges–Lehmann estimator-based localization method for NLOS environments and compare them with the existing robust localization methods such as the Taylor-series ML estimator using the median (hereafter abbreviated as the TML), M-estimator, and JMAP-ML estimator. The root mean squared error (RMSE) average of the proposed methods was similar or slightly superior to that using the M-estimator. The localization accuracy of the TML method is superior to that of the proposed methods. However, the proposed methods have an advantage in that they are the closed-form method. Also, the proposed methods outperform the JMAP-ML estimator in the above of the certain NLOS level if the contamination ratio is lower than the breakdown point (BP).

The organization of this paper is as follows. Section 2 explains the source localization problem to be solved in this paper. In Section 3, the details of the existing localization methods using the robust statistics are dealt with. The proposed location methods based on the α-trimmed mean and Hodges–Lehmann estimator are addressed in Section 4. To verify the robustness of the proposed method, the analysis of the BP, influence function (IF) and the asymptotic distribution of the proposed estimators are performed in Section 5. The estimation performance of the proposed methods is evaluated via simulation results in Section 6, comparing them with that of the existing algorithms. Lastly, Section 7 presents the conclusions.

2. Problem formulation

The TOA source localization method is aiming at finding the position of a source by using multiple circles whose centers represent the positions of sensors. The degradation of the quality of the TOA information is severe in the urban area or the indoor environment. In the NLOS source localization context, the measurement equation is represented as

$$r_{i,j} = d_i + n_{i,j} = \sqrt{(x-x_i)^2+(y-y_i)^2} + n_{i,j},$$
$$n_{i,j} \sim (1-\epsilon)N(0,\sigma_1^2)+\epsilon N(\mu_2,\sigma_2^2), \quad i=1,2,\ldots,M,\ j=1,2,\ldots,P \tag{1}$$

with M and P denoting the number of sensors and number of samples in the ith sensor. Also, $r_{i,j}$ is the measured distance between the source and the ith sensor at the jth sampling and d_i is the true distance between the source and the ith sensor. The measurement noise $n_{i,j}$ is modeled as a Gaussian mixture distribution in which the LOS noise is distributed according to $N(0,\sigma_1^2)$ with a probability $(1-\epsilon)$ or the NLOS noise distributed by $N(\mu_2,\sigma_2^2)$ with a probability of ϵ $[0<\epsilon<1$ is the fraction of contamination which has a small number (typically smaller than 0.1)] [24–26]. Also, $[x,y]^T$ is the true source position and $[x_i,y_i]^T$ is the position of the ith sensor. Note that, throughout this paper, a lowercase boldface letter denotes a vector, an uppercase boldface letter indicates a matrix and the superscript T signifies the vector/matrix transpose. The aim of this paper is to find the source position for which the RMSE of the position estimate is minimized.

3. Review of the existing NLOS TOA localization methods

3.1. Least squares (LS) method

Squaring (1) and rearranging yield the following equation:

$$x_i x + y_i y - 0.5R + m_{i,j} = 0.5(x_i^2+y_i^2-r_{i,j}^2), \quad i=1,2,\ldots,M,$$
$$j=1,2,\ldots,P \tag{2}$$

which can be simply represented in a matrix form as

$$\mathbf{Ax}+\mathbf{q}_j\mathbf{b}_j, \quad j=1,\ldots,P. \tag{3}$$

Note that

$$\mathbf{A}=\begin{pmatrix} x_1 & y_1 & -0.5 \\ \vdots & \vdots & \vdots \\ x_M & y_M & -0.5 \end{pmatrix}, \quad \mathbf{b}_j=\frac{1}{2}\begin{pmatrix} x_1^2+y_1^2-r_{1,j}^2 \\ \vdots \\ x_M^2+y_M^2-r_{M,j}^2 \end{pmatrix},$$
$$\mathbf{q}_j=[m_{1,j},\ldots,m_{M,j}]^T, \quad m_{i,j}=-d_i n_{i,j}-\frac{1}{2}n_{i,j}^2,$$
$$\mathbf{x}=[x\ y\ R]^T, \quad i=1,\ldots,M,\ j=1,\ldots,P,\ R=x^2+y^2 \tag{4}$$

where x_i, y_i is the position of the ith sensor, and $r_{i,j}$ is the measured distance between the source and the ith sensor at the jth sampling. The LS location estimate is obtained by minimizing the squared error sum as given by

$$\hat{\mathbf{x}}=(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\left\{\frac{1}{P}\sum_{j=1}^{P}\mathbf{b}_j\right\}. \tag{5}$$

The LS method has an efficiency when no outlier exists. However, when there are outliers, its estimation error drastically increases because it minimizes the squared error sum. The LS estimator relaxes the constraint $R=x^2+y^2$ when determines the position estimates and therefore leads to an overparameterization that may produce geometrically inconsistent results $\hat{R}\neq\hat{x}^2+\hat{y}^2$ in poor signal conditions.