Signal Processing 111 (2015) 170–178

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Localization of coherent signals without source number knowledge in unknown spatially correlated Gaussian noise ☆

Cheng Qian [a], Lei Huang [a,*], Yuhang Xiao [a], H.C. So [b]

[a] Department of Electronic and Information Engineering, Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen, China
[b] Department of Electronic Engineering, City University of Hong Kong, Hong Kong

ARTICLE INFO

Article history:
Received 9 June 2014
Received in revised form
1 November 2014
Accepted 6 December 2014
Available online 15 December 2014

Keywords:
Direction-of-arrival (DOA) estimation
Fourth-order cumulant
Coherent signals
Joint diagonalization
Toeplitz matrix

ABSTRACT

A new direction-of-arrival estimator for coherent signals in spatially correlated noise is devised in this paper. By constructing a set of fourth-order cumulant based Toeplitz matrices, the coherent signals can be decorrelated. Moreover, by utilizing the joint diagonalization structure of these Toeplitz matrices, a new cost function that does not require any *a priori* information of the source number is developed. Numerical examples are provided to demonstrate the effectiveness of the proposed approach.

1. Introduction

Direction finding using a sensor array is an important task in many applications, such as radar [1], sonar [2] and wireless communications [3]. Numerous direction-of-arrival (DOA) estimators have been proposed in the literature. Among them, subspace based DOA estimation methods, such as ESPRIT [4–6] and MUSIC [7–9], provide an excellent solution to this problem when the following three assumptions are satisfied:

A1 The number of sources is known *a priori*.
A2 The sources are mutually uncorrelated or partially correlated.
A3 The noise is spatially uncorrelated white noise, i.e., the covariance matrix is proportional to the identity matrix.

If any one of the above assumptions does not hold, this kind of techniques may suffer serious performance degradation.

As a matter of fact, the source number is usually unknown to the receiver in practice. To circumvent this issue, various source enumeration approaches have been suggested. Akaike information criterion (AIC) [10,11] and minimum description length (MDL) [12–15] are the most popular methods to estimate the number of sources. However, when the sample size is small and the signal-to-noise ratio (SNR) is low, they might not provide correct estimate of the source number. Although numerous modified algorithms have been proposed, the correct detection probability is still low in extreme conditions, especially when the noise property is unknown [13].

The **A2** cannot be satisfied in practice due to multipath propagation, which leads to many coherent components

☆ The work described in this paper was in part supported by the NSFC/RGC Joint Research Scheme sponsored by the Research Grants Council of Hong Kong and the National Natural Science Foundation of China (Project No.: N-CityU 104/11, 61110229/61161160564), by the National Natural Science under Grant 61222106 and Grant 61171187 and by the Shenzhen Kongqie Talent Program under Grant YFZZ20111013.
* Corresponding author.
E-mail addresses: chengqian@hitsz.edu.cn (C. Qian), dr.lei.huang@ieee.org (L. Huang), yuhangxiao@outlook.com (Y. Xiao), hcso@ee.cityu.edu.hk (H.C. So).

http://dx.doi.org/10.1016/j.sigpro.2014.12.005
0165-1684/

among the received data. Under such a case, the source covariance matrix is rank deficient, which in turn makes subspace based techniques to suffer serious performance degradation. Spatial smoothing (SS) technique and its variants [16–18] have been proposed to handle the coherent signals. They use a preprocessing scheme that first partitions the total array into subarrays and then averages the subarray output covariance matrices to make the source covariance matrix to be full rank, enabling the subspace based algorithms to work properly.

It is well known that most of the DOA estimation techniques are sensitive to the noise model [19,20] because they implicitly assume spatially uncorrelated noise. The spatially correlated noise can be easily handled by pre-whitening [21] provided that its covariance matrix is known *a priori*. However, in practice, since the array response and noise covariance are often computed from limited observations, an accurate covariance structure is often not available. The technique suggested in [22] is based on a parametric model to determine the noise covariance matrix, which allows the signal and noise parameters to be estimated simultaneously. In [23], a maximum likelihood (ML) based DOA estimation method has been proposed. It uses a set of sparse sensor arrays with multiple widely separated subarrays to make sensor noise uncorrelated between different subarrays, and then applies the ML method to estimate the DOAs.

The conventional array processing techniques are usually based on the covariance matrix which corresponds to the second-order statistics of the received signals. Indeed, the received signals are often non-Gaussian in practice, e.g., the BPSK, QPSK and QAM modulated signals. For non-Gaussian signals, the second-order statistics are not sufficient to characterize their statistical behavior. Higher-order moments are preferred to explore the non-Gaussianity of the signals. A number of higher-order statistics based DOA estimators have been proposed in the literature. Zeng et al. [24] use a set of fourth-order cumulant matrices to devise a new DOA estimation method that does not need to know the source number. However, it cannot deal with the coherent signals. Doğan and Mendel [25] use the higher-order cumulant to generate virtual aperture and then devise a virtual-ESPRIT algorithm (VESPA) which utilizes the information between virtual and actual sensors to solve the problem of joint array calibration and DOA estimation. The VESPA works properly for uncorrelated signals. For coherent signals, an extended VESPA (EVESPA) [26] has been proposed. Since each signal eigenvector associated with a group of coherent signals contains all the DOA information of these signals, the EVESPA applies the SS technique to this signal eigenvector to construct a full-rank signal subspace. Then it utilizes the root-MUSIC method to yield the DOA estimates. In [27], a higher-order cumulant MUSIC algorithm has been developed. This scheme can correctly work for the underdetermined case where the number of signals is larger than the number of sensors, but it is unable to handle the coherent signals.

To overcome the aforementioned shortcomings of the existing subspace based DOA estimators, we propose a new DOA estimator that is based on the joint diagonalization of a set of Toeplitz matrices. In this paper, we consider a centro-symmetric uniform linear array (ULA) of $N=2M+1$ sensors with half-wavelength interelement spacing. By employing the fourth-order cumulant technique, the Gaussian noise can be eliminated. Moreover, $(2M+1)(M+1)$ different cumulant matrices can be formed, allowing us to handle the coherent signal issue. In particular, each row of a cumulant matrix is used to construct a Toeplitz matrix to decorrelate the coherent signals. Since these Toeplitz matrices share the joint diagonalization structure, a new cost function that does not need *a priori* information of source number is devised. A new spatial spectrum is then obtained where the DOAs are estimated via a *one-dimensional* search.

The remainder of the paper is organized as follows. Section 2 describes the direction finding problem and introduces the mathematical assumptions. The definition of the fourth-order cumulant, calculation of the Toeplitz covariance matrix and joint diagonalization based DOA estimation method are presented in Section 3. Simulation results are given in Section 4. Finally, conclusions are drawn in Section 5.

Throughout this paper, we use boldface uppercase letters to denote matrices, boldface lowercase letters for column vectors, and lowercase letters for scalar quantities. Superscripts $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$, $(\cdot)^{-1}$ and $(\cdot)^\dagger$ represent transpose, complex conjugate, conjugate transpose, inverse and pseudo-inverse, respectively. The operator $\mathbb{E}\{a\}$ is the expected value of a, $\mathbf{0}$ is the zero matrix, $\mathbf{I}_M$ is the $M\times M$ identity matrix and $\mathbf{J}_M$ is a $M\times M$ exchanging matrix with its anti-diagonal being one and zero elsewhere. The $\mathbb{C}$ denotes the set of complex numbers. Furthermore, $\|\cdot\|$ represents the Euclidean norm of a vector.

2. Problem formulation

Consider a ULA with $N=2M+1$ isotropic sensors shown in Fig. 1. There are P ($P\le M+1$) narrowband source signals impinging on the array from distinct directions $\{\theta_1,\ldots,\theta_P\}$ in the far field and the first K signals are mutually coherent and the others are uncorrelated and independent of the first K signals. Taking the first signal $s_1(t)$ as reference, the kth coherent signal becomes

$$s_k(t)=\beta_k e^{j\delta\phi_k}s_1(t),\quad k=2,\ldots,K \tag{1}$$

where β_k is the amplitude fading factor and $\delta\phi_k$ is the phase change. Since the values of $\delta\phi_k$ will not affect the coherence between the signals, without loss of generality, we set $\delta\phi_k=0, k=2,\ldots,K$. Then the signals arriving at the mth sensor at time t can be expressed as

$$x_m(t)=\sum_{i=1}^{P}s_i(t)e^{-j2\pi m\sin(\theta_i)\Delta/\lambda}+n_m(t)$$

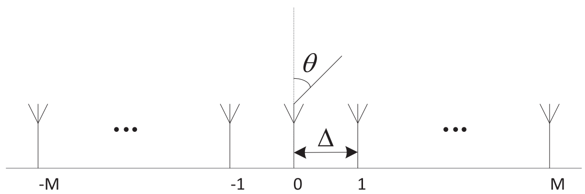

Fig. 1. Symmetric ULA model.