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On geometric upper bounds for positioning algorithms in wireless sensor networks

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ABSTRACT

This paper studies the possibility of upper bounding the position error for range-based positioning algorithms in wireless sensor networks. In this study, we argue that in certain situations when the measured distances between sensor nodes have positive errors, e.g., in non-line-of-sight (NLOS) conditions, the target node is confined to a closed bounded convex set (a feasible set) which can be derived from the measurements. Then, we formulate two classes of geometric upper bounds with respect to the feasible set. If an estimate is available, either feasible or infeasible, the position error can be upper bounded as the maximum distance between the estimate and any point in the feasible set (the first bound). Alternatively, if an estimate given by a positioning algorithm is always feasible, the maximum length of the feasible set is an upper bound on position error (the second bound). These bounds are formulated as nonconvex optimization problems. To progress, we relax the nonconvex problems and obtain convex problems, which can be efficiently solved. Simulation results show that the proposed bounds are reasonably tight in many situations, especially for NLOS conditions.

1. Introduction

Recent advances in technology have instigated the use of tiny devices as sensors in large distributed wireless sensor networks (WSNs). A sensor device is capable to sense its environment for monitoring, controlling, or tracking purposes for both civil and military applications [1]. Due to drawbacks in using GPS for WSNs, extracting the position information from the network, also called localization, has been extensively studied in the literature [2,1,3–6]. It is commonly assumed that there are a number of fixed reference sensors, also called anchors, whose positions are *a priori* known, e.g., by using GPS receivers [7]. To find the position of other sensor nodes at unknown positions, henceforth called target nodes, it is assumed that there are some types of measurements, e.g., time-of-arrival, angle-of-arrival, or received signal strength, taken between sensor nodes [1].

During the last decades, various positioning algorithms have been proposed in the literature. Different positioning approaches can be categorized based on various factors [8]. For instance, as long as an accurate model of measurements and the statistics of the measurement errors are known, classic estimators, e.g., the maximum likelihood (ML) and the least squares (LS) approaches, can be

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employed successfully to solve the positioning problem. When the distribution of the measurement errors is unknown or the computational complexity of classic estimators is too high, a number of simple techniques can be applied to the problem. For example, suboptimal algorithms such as semidefinite programming (SDP) [9] or closed-form linear least squares (LLS) [10,11] have been successfully applied to the positioning problem. In one class of suboptimal algorithms based on a geometric interpretation, the authors of [12,13] formulated the positioning problem as a convex feasibility problem (CFP) and applied the well-known orthogonal projection onto convex sets (POCS) approach to solve the problem. This method turns out to be robust against non-line-of-sight (NLOS) conditions [14]. POCS was previously studied for the CFP and has found applications in several research fields [15,16].

Positioning algorithms can be evaluated based on different performance metrics such as complexity, accuracy, and coverage [8]. In the literature one way to assess the positioning algorithms is to evaluate the position error, defined as the Euclidian norm of the difference between the position estimate and the true position. There are a number of techniques to evaluate the performance of an algorithm based on the position error. For instance, a lower bound on the mean square position error is a common metric [17,18]. There exist a number of such lower bounds for the positioning algorithms in the literature. For example the Cramér–Rao lower bound (CRLB), which gives a lower bound on the variance of any unbiased estimator, can be computed if the probability density function (PDF) of the measurement error is known and satisfies some regularity conditions [19]. Generally, different benchmarks in the literature are used to *statistically* assess a positioning algorithm, which implies that the error in a single position estimate cannot be characterized in a deterministic fashion.

Besides a lower bound on the position error, in some applications it may be useful to know the worst-case behavior of the position error. Such knowledge may be useful not only for evaluation of different services provided by WSNs but also for design and resource management [1,20]. Similarly in evaluation of the worst-case position error, we may be interested in assessing a single point estimate. As an example consider Fig. 1, which shows how a nontrivial (i.e., finite) upper bound on the position error can be used by a traffic safety application to decrease collisions between vehicles. If an estimate of a vehicle and a nontrivial upper bound on the position error are available, we can define an area in which the vehicle is certainly located, e.g., a disc centered at the position estimate and with a radius equal to the upper bound on the position error. Such an estimate can be obtained in every vehicle, for instance, by measuring the distance between the vehicle and a number of fixed nodes (at known positions) along the road. The estimate of cars' positions and upper bounds on the position errors can be exchanged between vehicles. By this approach, we may be able to decrease the number of collisions between vehicles. In general, computing the position error might be difficult since the true position is unknown, but one may be able to derive an upper bound on the position error. To the best of our knowledge, there is no specific work in the literature on deriving upper bounds on the position error. In this study, we aim at tackling this subject in a geometric framework.

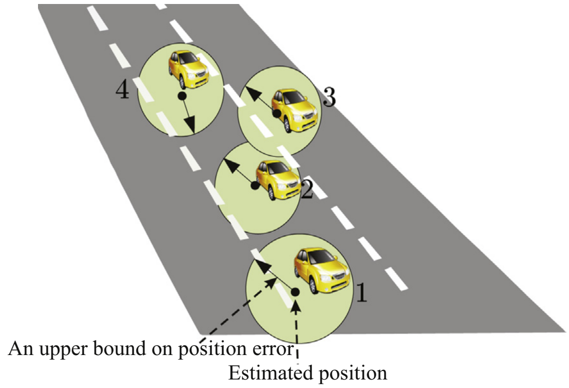

Fig. 1. An example of the application of an upper bound on the position error for traffic safety. A solid circle defines the area in which a vehicle definitely lies. In this figure based on an upper bound on the position error, car 2 and 3 might collide.

In general, the concept of an upper bound on the position error (or any estimation error) seems to be questionable. In fact, it is not clear if it is meaningful to study upper bounds, since the position error can, in general, be arbitrarily large. In this study, however, we argue that in some practical situations, the position error is finite and can be upper bounded. For instance, if a target node position belongs to a closed bounded set (a feasible set), an upper bound on the position error can be computed from the feasible set. For example, for distance-based positioning, if measurement errors are assumed to be positive, a convex set including the target node can be defined from measurements. The feasible set, in which the target node is located, is the intersection of a number of balls (in a 3-dimensional network) or discs (in a 2-dimensional network) centered at the position of reference nodes [21]. The assumption of positive measurement errors is fulfilled in some scenarios. For instance, in NLOS conditions, the measured distances are often much larger than the actual distances. For practical ranging using UWB, it has been observed that the measurement errors tend to be positive, even for line-of-sight (LOS) scenarios [22]. It should be noted that the measurement error, in general, can be negative as well, meaning the intersection no longer contains the location of the target node and the bounding technique may not work properly. In such scenarios, one can, e.g., modify the measurements and obtain a new set of distance measurements that are larger than the actual distances. For example, if a reasonable lower bound on negative measurement errors is available, then we can enlarge the measurements with the absolute value of the lower bound and obtain a set of measurements with positive errors. Now, assuming a closed bounded (compact) convex set derived from distance measurements having positive errors, a position estimate given by an algorithm can be either feasible or infeasible with respect to the feasible set. If an estimate is available (feasible or infeasible), it is reasonable to define the maximum distance from the estimate to any point in the