Signal Processing 95 (2014) 32–42

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Design of allpass variable fractional delay filter with signed powers-of-two coefficients

Changjun Yu*, Kok Lay Teo, Hai Huyen Dam

Department of Mathematics and Statistics, Curtin University, WA, Australia

ARTICLE INFO

Article history:
Received 6 March 2013
Received in revised form
30 May 2013
Accepted 17 August 2013
Available online 25 August 2013

Keywords:
Allpass variable fractional delay filter
Integer programming
Exact penalty function method

ABSTRACT

This paper investigates the optimal design of allpass variable fractional delay (VFD) filters with coefficients expressed as sums of signed powers-of-two terms, where the weighted integral squared error is the cost function to be minimized. The design can be classified as an integer programming problem. To solve this problem, a new procedure is proposed to generate a reduced discrete search region to decrease the computational complexity. A new exact penalty function method is developed to solve the optimal design problem for allpass VFD filter with signed powers-of-two coefficients. Design examples show that the proposed method can achieve a higher accuracy when compared with the quantization method.

1. Introduction

Digital filters with tunable fractional phase-delay or fractional group delay, referred to as variable fractional delay (VFD) filters, are useful in various signal processing applications, such as timing offset recovery in digital receivers, comb filter design, sampling rate conversion, speech coding, time delay estimation, one-dimensional digital signal interpolation and image interpolation [1–5]. For finite impulse response (FIR) based VFD filters, a natural optimization problem can be formulated. Then, an approximate problem with the desired characteristics being achieved is constructed. For this approximate problem, it can be easier solved (see [5–7]). The design of allpass VFD filters is more involved [9–11]. The key advantage of allpass VFD filters is that they can achieve higher design accuracy than FIR filters, yielding smaller frequency response errors for applications that require unity gain. However, since an allpass VFD filter has infinite impulse response, adjusting its coefficients will cause undesirable transient response. In general, the transient response depends on the magnitude of the input signal, the number of the coefficients, and the speed of the impulse responses decay. Efforts to minimize the undesirable transient behavior can be found in [13].

In [14], the design of allpass VFD filters with least squares and minimax group delay errors is investigated. The design of minimax phase error allpass VFD filters is discussed in [15]. In [1,10], the design of an allpass VFD filter with minimum integral squared error is developed. The obtained filters might have large deviation from the desired response, especially at the cutoff frequencies. In addition, several restrictions are required for the VFD filter specification. In [16], the minimax optimization problem is solved by fixing the coefficient of the denominator and iteratively updating coefficients of the numerator. The minimax allpass filters may, however, have large integral squared error. Also the approach is only applicable for the design of allpass VFD filters with infinite precision coefficients.

For ease in practical implementation, many approaches have been proposed to design low complexity filters. One of the most common strategies is to optimize the coefficients in signed powers-of-two (SPT) space, where each coefficient is written as a sum of a given number of SPT terms. This type of problem can be transformed into traditional integer programming problem, which can, in principle, be solved by many existing optimization methods (see, for example, [8,24]).

* Corresponding author. Tel.: +61 425841018.
E-mail address: yuchangjun@126.com (C. Yu).

0165-1684/$ - see front matter
http://dx.doi.org/10.1016/j.sigpro.2013.08.005

Simulated annealing methods [25] and genetic algorithm [26] have also been applied for the design of digital filters with SPT coefficients. In [12], a three-step procedure is given for finding the multiplierless coefficient representations for adjustable fractional delay allpass filters. However, the global minimizer could only be obtained by an exhausted search in the search region, which is not applicable for large scale design problem. In [8], a two-step algorithm is developed for the design of FIR filters with signed-powers-of-two coefficients. The first step is a local search, where the steepest descent algorithm is used to find a local minimizer. Then, in the second step, a discrete filled function method is used so that the current local minimizer becomes a local maximizer and from which a better minimizer can be found. Among the existing methods, the quantization method [20–23] is most commonly used for finding a signed powers-of-two solution as it is simple and easy to implement.

In this paper, we investigate the design of allpass VFD filters with signed powers-of-two coefficients and a least square criterion. By using the approximation scheme obtained in [1], the objective function is approximated by a quadratic cost function which has a unique infinite precision optimal solution. Based on this optimal solution, a good search region containing the global solution is developed by using a two-step scheme. Then, a new exact penalty function method is proposed to solve the quadratic integer optimization problem defined in the obtained search region. To demonstrate the effectiveness of the proposed method, some design examples are solved.

The rest of the paper is organized as follows. The problem formulation is given in Section 2. Before we present our proposed solution method in Section 4, a computational procedure to construct appropriate reduced search region is given in Section 3. Simulation results are discussed in Section 5 and finally some concluding remarks are made in Section 6.

2. Problem formulation

Consider the design of an allpass VFD filter with the desired frequency response $H_d(\omega, p)$ given by

$$H_d(\omega, p) = e^{-j(N+p)\omega} \tag{1}$$

where N is a fixed group-delay, p denotes the fractional group-delay which is continuously varied in the range $\mathcal{P} = [p_0, p_0+1]$, and $\omega \in \Omega = [0, \alpha\pi]$ with $\alpha > 0$. Each coefficient of the allpass filter can be expressed as a polynomial of p given below:

$$a_n(p) = \sum_{m=1}^{M} c_{n,m} p^m, \quad 1 \le n \le N. \tag{2}$$

For ease in practical implementation, the coefficients $c_{n,m}$ are expressed in the form of sum of signed-powers-of-two terms given as

$$c_{n,m} = \sum_{i=1}^{b} d_{i,n,m} 2^{-i} \tag{3}$$

where $d_{i,n,m} \in \{-1, 0, 1\}$, $i = 1, \ldots, b, b$ denotes the number of bits of the word length, $n = 1, \ldots, N$, and $m = 1, \ldots, M$.

Define

$$\mathcal{Z}_b = \left\{ \sum_{i=1}^{b} d_{i,n,m} 2^{-i} | d_{i,n,m} \in \{-1, 0, 1\} \right\},$$

and

$$C = \{c_{n,m} | n = 1, \ldots, N, \ m = 1, \ldots, M\}.$$

Furthermore, let N_1 denote the total allowable number of signed-powers-of-two terms used. Then, the following constraint must be satisfied:

$$\sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{i=1}^{b} |d_{i,n,m}| \le N_1. \tag{4}$$

The frequency response of the allpass filter is given by

$$\begin{aligned} H(C, \omega, p) &= \frac{a_n(p) + \cdots + a_1(p) e^{-j(N-1)\omega} + e^{-jN\omega}}{1 + a_1(p) e^{-j\omega} + \cdots + a_n(p) e^{-jN\omega}} \\ &= e^{-jN\omega} \frac{1 + \sum_{n=1}^{N} a_n(p) e^{jn\omega}}{1 + \sum_{n=1}^{N} a_n(p) e^{-jn\omega}} \\ &= e^{-jN\omega} \frac{1 + \sum_{n=1}^{N} \sum_{m=1}^{M} c_{n,m} p^m e^{jn\omega}}{1 + \sum_{n=1}^{N} \sum_{m=1}^{M} c_{n,m} p^m e^{-jn\omega}}. \end{aligned} \tag{5}$$

Let $R(C, \omega, p) = 1 + \sum_{n=1}^{N} \sum_{m=1}^{M} c_{n,m} p^m e^{jn\omega}$. Then, (5) can be rewritten as

$$H(C, \omega, p) = e^{-jN\omega} \frac{R(C, \omega, p)}{R^*(C, \omega, p)}$$

where $R^*(C, \omega, p)$ denotes the complex conjugate of $R(C, \omega, p)$.

Let

$$e_H(\omega, p) = |H(C, \omega, p) - H_d(\omega, p)|.$$

Then, the design objective is to choose the coefficients $c_{n,m}$ in the form of (3) such that

$$\int_{p_0}^{p_0+1} \int_0^{\alpha\pi} W(\omega, p) e_H(\omega, p)^2 \, d\omega \, dp \tag{6}$$

is minimized, subject to constraint (4), where $W(\omega, p)$ is a positive weighting function. It is assumed that $W(\omega, p)$ is separable, i.e.,

$$W(\omega, p) = W_1(\omega) W_2(p),$$

where $W_1(\omega)$ and $W_2(p)$ are piecewise constant functions. This problem is referred to as Problem (P).

Problem (P) is a constrained nonlinear integer programming problem. Noting that, for each $n = 1, \ldots, N$ and $m = 1, \ldots, M$, $c_{n,m}$ has at most $2^{b+1} - 1$ options. This is enormously large, and hence making the problem extremely difficult to solve. A natural way to reduce the complexity is to reduce the number of options for each $c_{n,m}$. Since the objective function of Problem (P) is quadratic, the discrete points in the neighborhoods of the infinite precision optimal solution of Problem (P) are good choices for each $c_{n,m}$. Thus, we propose to solve Problem (P) in three stages as follows:

(I) Obtain the infinite precision optimal solution for Problem (P).
(II) Find a reduced search region around the minimizer obtained in Stage I.
(III) Find a point that minimizes the objective function (6) within the region obtained in Stage II.