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Similar sensing matrix pursuit: An efficient reconstruction algorithm to cope with deterministic sensing matrix

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ABSTRACT

Deterministic sensing matrices are useful, because in practice, the sampler has to be a deterministic matrix. It is quite challenging to design a deterministic sensing matrix with low coherence. In this paper, we consider a more general condition, when the deterministic sensing matrix has high coherence and does not satisfy the restricted isometry property (RIP). A novel algorithm, called the similar sensing matrix pursuit (SSMP), is proposed to reconstruct a *K*-sparse signal, based on the original deterministic sensing matrix. The proposed algorithm consists of off-line and online processing. The goal of the off-line processing is to construct a similar compact sensing matrix containing as much information as possible from the original sensing matrix. The similar compact sensing matrix has low coherence, which guarantees a perfect reconstruction of the sparse vector with high probability. The online processing begins when measurements arrive, and consists of rough and refined estimation processes. Results from our simulation show that the proposed algorithm obtains much better performance while coping with a deterministic sensing matrix with high coherence compared with the subspace pursuit (SP) and basis pursuit (BP) algorithms.

1. Introduction

Compressed sensing has received considerable attention recently, and has been applied successfully in diverse fields, e.g. image processing [1,2], underwater acoustic communication [3], wireless communication [4] and radar systems [5–9]. The central goal of compressed sensing is to capture attributes of a signal using very few measurements. In most work to date, this broader objective is exemplified by the important special case in which a K-sparse vector $\mathbf{x} \in R^N$ (with N large) is to be reconstructed from a small number M of linear measurements with $K < M < N$. K-sparse signals are the signals that can be represented by K significant coefficients over an N-dimensional basis. This can be compactly described via

$$\mathbf{y} = \mathbf{\Phi x} + \mathbf{e}. \qquad (1)$$

Here, $\mathbf{y} \in R^M$ denotes a measurement vector, $\mathbf{\Phi}$ represents an $M \times N$ sensing matrix, and $\mathbf{e}$ is an $M \times 1$ noise vector. The two fundamental questions in compressed sensing are as follows: how to construct suitable sensing matrix $\mathbf{\Phi}$, and how to recover the K-sparse vector $\mathbf{x}$ from the measurement vector $\mathbf{y}$ efficiently. Tables A1 and A2 in Appendix A list notations for variables used in the paper.

In early work of compressed sensing, the entries of the sensing matrix $\mathbf{\Phi}$ are generated by an independent and identically distributed (i.i.d) Gaussian or Bernoulli process, or from random Fourier ensembles [10–12]. In general, the exact solution to the above second question is shown to be an NP-hard problem [13,14]. If the number of samples (M) exceeds the lower bound of $M > O(K \log(N/K))$, l_1 minimization (e.g. BP algorithm) can be performed instead of the exact l_0 minimization with the same solution for almost all the possible inputs [14]. An alternative approach to sparse signal recovery is based on the idea of iterative greedy pursuit, and tries to approximate the solution to

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l_0 minimization directly. The greedy algorithms include matching pursuit (MP) [15], orthogonal matching pursuit (OMP) [16], regularized OMP (ROMP) [17], stagewise OMP (StOMP) [18], SP [19], compressive sampling matching pursuit (CoSaMP) [20] and backtracking-based matching pursuit (BAOMP) [21], etc. The reconstruction complexity of these approximate algorithms is significantly lower than that of BP algorithm.

In compressed sensing, one of the well-studied conditions on the sensing matrix, which guarantees stable recovery for a number of reconstruction algorithms, is the RIP [13,14]. If a sensing matrix $\mathbf{\Phi}$ whose column vectors have unit norm and satisfies

$$(1-\delta_K)\|\mathbf{x}\|_2^2 \le \|\mathbf{\Phi x}\|_2^2 \le (1+\delta_K)\|\mathbf{x}\|_2^2 \tag{2}$$

for all possible K-sparse vectors with restricted isometry constant (RIC) δ_K, then $\mathbf{\Phi}$ is said to obey K-RIP with δ_K. The RIP with suitable constant δ_K guarantees perfect reconstruction [13,14], but it is very hard to check whether a sensing matrix satisfies RIP or not.

Coherence, the maximal correlation between two columns in a sensing matrix, is also a well-known performance measure for sensing matrices. For a matrix $\mathbf{\Phi}$ with columns $\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \dots, \boldsymbol{\varphi}_N$, the coherence of $\mathbf{\Phi}$ is defined as

$$\mu(\mathbf{\Phi}) = \max_{1 \le i,j \le N \text{ and } i \ne j} \frac{|\boldsymbol{\varphi}_i^T \boldsymbol{\varphi}_j|}{\|\boldsymbol{\varphi}_i\| \cdot \|\boldsymbol{\varphi}_j\|}. \tag{3}$$

Coherence plays a central role in the sensing matrix construction, because small coherence implies the RIP [22].

In this paper, we are interested in deterministic sensing matrices. Deterministic sensing matrices are useful because in practice, the sampler has to be a deterministic matrix. Although random matrices perform quite well on the average, there is no guarantee that a specific realization works. For the deterministic approaches, the Vandermond matrices seem to be good options, since any K columns of a $K \times N$ Vandermond matrix are linearly independent. However, when N increases, the constant δ_K rapidly approaches 1 and some of the $K \times K$ submatrices become ill-conditioned [23]. A connection between the coding theory and sensing matrices is established in [24] where second order Reed–Muller codes are used to construct bipolar matrices. However, they lack a guarantee on the RIP order. In [25], the authors propose a series of deterministic sensing matrices, the binary, bipolar, and ternary compressed sensing matrices which satisfy the RIP condition.

The key concept of coherence is extended to pairs of orthonormal bases. This enables a new choice of the sensing matrices: one simply selects an orthonormal basis that is incoherent with the sparsity basis, and obtains measurements by selecting a subset of the coefficients of the signal in the chosen basis [26]. This approach has successful applications in radar systems [9,27], where an additional sensing matrix H is introduced and the received signal is compressed further by making nonadaptive, linear projections of the direct data sampled at the Nyquist frequency. However, neither of these algorithms mention the hardware implementation of the additional sensing matrix, which is very complex and expensive.

In practice, it is challenging to design a deterministic sensing matrix having low coherence. In this paper, we consider a more general condition when the deterministic sensing matrix has high coherence and does not satisfy the RIP condition. A novel algorithm, called the SSMP algorithm, is proposed to reconstruct the K-sparse signal based on the original deterministic sensing matrix. The proposed algorithm consists of two parts: the off-line processing and the online processing. The goal of the off-line processing is to construct a similar compact sensing matrix with low coherence, which contains as much information as possible from the original sensing matrix. The online processing begins when the measurements arrive, which consists of a rough estimation process and a refined estimation process. In the rough estimation process, an SP algorithm is used to find a rough estimate of the true support set, which contains the indices of the columns that contribute to the original sparse vector. Three kinds of structures of the estimated support set are considered, and three individual refined estimation processes are carried out under these three conditions. We observe from simulation results that the proposed algorithm obtains much better performance when coping with the deterministic sensing matrix with high coherence compared with the SP and BP algorithms.

The paper is organized as follows. Section 2 introduces the proposed similar sensing matrix pursuit algorithm. Section 3 presents simulation results, and Section 4 summarizes conclusions.

2. Similar sensing matrix pursuit (SSMP) algorithm

Recently, algorithms used to cope with deterministic sensing matrices focus on designing a sensing matrix which satisfies the RIP condition (or with low coherence). However, in practice it is challenging to design a deterministic sensing matrix having a very small restricted isometry constant (coherence). In this paper, we consider a more general condition when a deterministic sensing matrix has high coherence and does not satisfy the RIP condition. We concentrate in developing a novel reconstruction algorithm rather than building a deterministic sensing matrix with low coherence. A novel algorithm called the SSMP is proposed to reconstruct the K-sparse signal, based on the original deterministic sensing matrix.

This section introduces the proposed SSMP algorithm. First, the key component of the proposed algorithm, the similar compact sensing matrix, is introduced in Section 2.1, and then the complete algorithm is described in Section 2.2. The complexity analysis of the proposed algorithm is presented in Section 2.3.

2.1. Construction of the similar compact sensing matrix

The construction process of the similar compact sensing matrix is based on the similarity analysis of the original sensing matrix. In this paper, similarity is defined as the absolute and normalized inner product between any two different columns of the original sensing matrix $\mathbf{\Phi}$:

$$\lambda(\boldsymbol{\varphi}_i, \boldsymbol{\varphi}_j) = \frac{|\boldsymbol{\varphi}_i^T \boldsymbol{\varphi}_j|}{\|\boldsymbol{\varphi}_i\| \cdot \|\boldsymbol{\varphi}_j\|}, \quad 1 \le i,j \le N \quad \text{and} \quad i \ne j. \tag{4}$$