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Two new criteria for the realization of interfered digital filters utilizing saturation overflow nonlinearity

Choon Ki Ahn *

School of Electrical Engineering, Korea University, 145, Anam-ro, Seongbuk-gu, Seoul 136-701 Republic of Korea

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ABSTRACT

In this paper, an input-to-state stability (ISS) approach is used to derive two new criteria for the realization of fixed-point state-space and direct-form digital filters with saturation overflow nonlinearity and external interference via augmented Lyapunov functions. The two proposed realization criteria ensure ISS for external interference. Moreover, these criteria guarantee asymptotic stability without external interference. They take the form of linear matrix inequality (LMI) and, hence, are computationally tractable. Illustrative examples demonstrate the effectiveness of the two proposed criteria.

1. Introduction

When we implement digital filters using fixed-point arithmetic on a digital computer or on special-purpose digital hardware, the effects of nonlinearities (quantization and overflow) due to finite wordlength are unavoidable. If the total number of quantization steps is large, the effects of these nonlinearities can be regarded as decoupled, and thus, can be examined separately. The stability analysis of digital filters employing saturation overflow arithmetic has attracted the attention of many researchers [1–13]. However, most existing stability criteria for digital filters are only available under specific conditions. In unfavorable environments with parameter uncertainties or external interferences, these criteria will be of little use. In order to handle external interferences, Ahn recently proposed some new stability criteria for digital filters based on $\mathcal{H}_\infty$ norm, l_2–l_∞ norm, and induced l_∞ norm [14–20]. However, these criteria are somewhat conservative because they were derived based on a simple Lyapunov function.

Real physical systems are constantly corrupted by disturbances or interferences. Thus, real physical systems are required not only to be stable, but also to have the property of input-to-state stability (ISS). The ISS concept, which was first introduced in [21], is an important approach for examining the stability of dynamical systems [21–23]. This approach can handle disturbances or interferences in dynamical systems using only the input–output information. Recently, the main results on the ISS and related notions were reviewed in [24]. At this point, a natural question arises: Can we obtain ISS criteria for digital filters with external interference? In the present paper, we answer this question. To the best of our knowledge, the ISS-based criteria of digital filters with saturation arithmetic and external interference have never been studied in the literature to date; these remain unresolved and challenging tasks.

In this paper, we propose new ISS criteria for fixed-point state-space and direct-form digital filters with saturation arithmetic and external interference via augmented Lyapunov functions. These two criteria provide a

* Tel.: +82 2 3290 4831.
E-mail addresses: hironaka@korea.ac.kr, hironaka@hanmail.net

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new contribution to the topic of stability analysis for digital filters and ensure that digital filters are asymptotically stable and input-to-state stable for external interference. These criteria can be represented by linear matrix inequalities (LMIs), which can be checked using standard numerical algorithms [25,26].

Our research is organized as follows. In Section 2, two new LMI-based ISS criteria for fixed-point state-space and direct-form digital filters are proposed. In Section 3, two numerical examples are given, and finally, conclusions are presented in Section 4.

2. ISS analysis for digital filters

A function $\gamma: R_{\geq 0} \to R_{\geq 0}$ is a $\mathcal{K}$ function if it is continuous, strictly increasing, and $\gamma(0)=0$. It is a $\mathcal{K}_\infty$ function if it is a $\mathcal{K}$ function and also $\gamma(s)\to\infty$ as $s\to\infty$. A function $\beta: R_{\geq 0}\times R_{\geq 0}\to R_{\geq 0}$ is a $\mathcal{KL}$ function if, for each fixed $k\geq 0$, the function $\beta(\cdot,k)$ is a $\mathcal{K}$ function, and for each fixed $s\geq 0$, the function $\beta(s,\cdot)$ is a continuous and decreasing function and $\beta(s,k)\to 0$ as $k\to\infty$. Then, a nonlinear system is said to be input-to-state stable if both a $\mathcal{K}$ function $\gamma(s)$ and a $\mathcal{KL}$ function $\beta(s,k)$ exist, such that, for each input U_k and each initial state X_0, the state X_k satisfies $\|X_k\| \leq \beta(\|X_0\|,k)+\gamma(\max_{0\leq\mu\leq k}\|U_\mu\|)$ for each $k\geq 0$.

2.1. ISS analysis for fixed-point state-space digital filters

The digital filter under consideration is described by

$$x(r+1)=f(y(r))+Gw(r), \tag{1}$$

$$y(r)=Ax(r), \tag{2}$$

where $f(y(r))=[f_1(y_1(r))\ f_2(y_2(r))\ \cdots\ f_n(y_n(r))]^T$, $x(r)=[x_1(r)\ x_2(r)\ \cdots\ x_n(r)]^T\in R^n$ is a state vector, $w(r)=[w_1(r)\ w_2(r)\ \cdots\ w_m(r)]^T\in R^m$ is an external interference, $A\in R^{n\times n}$ is the coefficient matrix, and $G\in R^{n\times m}$ is a known constant matrix. Now, we define $y(r)=[y_1(r)\ y_2(r)\ \cdots\ y_n(r)]^T\in R^n$ as an output vector. We assume that there is no interference in (2). The following saturation nonlinearities:

$$f_i(y_i(r))=\begin{cases}1 & \text{if } y_i(r)>1\\ y_i(r) & \text{if } -1\leq y_i(r)\leq 1\\ -1 & \text{if } y_i(r)<-1\end{cases} \tag{3}$$

are under consideration for $i=1,2,\ldots,n$. Note that the saturation nonlinearities are confined to the sector [0, 1], i.e.,

$$f_i(0)=0,\quad 0\leq\frac{f_i(y_i(r))}{y_i(r)}\leq 1,\quad i=1,2,\ldots,n. \tag{4}$$

In this subsection, we find a new LMI criterion such that the digital filter (1)–(2) with $w(r)=0$ is asymptotically stable ($\lim_{r\to\infty}x(r)=0$) and

$$\|x(r)\|\leq\beta(\|x(0)\|,r)+\gamma\left(\max_{0\leq\mu\leq r}\|w(\mu)\|\right) \tag{5}$$

for each $r\geq 0$.

Theorem 1. *If we assume that there exist symmetric positive definite matrices P, Q, S, R, a positive diagonal matrix M, and positive scalars δ_1, δ_2 such that $\Gamma_1<0$, where*

$$\Gamma_1=\begin{bmatrix}\delta_1A^TA+S-P & A^TM & 0 & 0\\ MA & P-Q-\delta_1I+\delta_2A^TA-2m & 0 & PG+\delta_2A^TAG\\ 0 & 0 & Q-\delta_2I & 0\\ 0 & G^TP+\delta_2G^TA^TA & 0 & \delta_2G^TA^TAG+G^TPG-R\end{bmatrix}, \tag{6}$$

then the digital filter (1)–(2) is input-to-state stable.

Proof. Consider the following Lyapunov function:

$$V(x(r))=x^T(r)Px(r)+f^T(Ax(r))Qf(Ax(r)), \tag{7}$$

which satisfies the following inequality:

$$\begin{aligned}\lambda_{\min}(P)\|x(r)\|^2 &\leq V(x(r))\\ &\leq\lambda_{\max}(P)\|x(r)\|^2+\lambda_{\max}(Q)\|f(Ax(r))\|^2\\ &\leq\lambda_{\max}(P)\|x(r)\|^2+\lambda_{\max}(Q)\|Ax(r)\|^2\\ &\leq\lambda_{\max}(P)\|x(r)\|^2+\lambda_{\max}(Q)\lambda_{\max}(A^TA)\|x(r)\|^2\\ &=(\lambda_{\max}(P)+\lambda_{\max}(Q)\lambda_{\max}(A^TA))\|x(r)\|^2,\end{aligned} \tag{8}$$

where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the maximum and minimum eigenvalues of the matrix. Along the trajectory of (1), we have

$$\begin{aligned}\Delta V(x(r)) &= V(x(r+1))-V(x(r))\\ &=[f(Ax(r))+Gw(r)]^TP[f(Ax(r))+Gw(r)]+f^T(Af(y(r))\\ &\quad+AGw(r))Qf(Af(y(r))+AGw(r))-x^T(r)Px(r)\\ &\quad-f^T(Ax(r))Qf(Ax(r))\\ &\quad=f^T(Ax(r))[P-Q]f(Ax(r))+f^T(Af(y(r))\\ &\quad+AGw(r))Qf(Af(y(r))+AGw(r))\\ &\quad+f^T(Ax(r))PGw(r)+w^T(r)G^TPf(Ax(r))\\ &\quad+w^T(r)G^TPGw(r)-x^T(r)Px(r)\\ &\quad+2f^T(Ax(r))M[Ax(r)-f(Ax(r))]\\ &\quad-2f^T(y(r))M[y(r)-f(y(r))].\end{aligned}$$

The condition (4) implies

$$f^T(Ax(r))f(Ax(r))=\|f(Ax(r))\|^2\leq\|Ax(r)\|^2=x^T(r)A^TAx(r) \tag{9}$$

and

$$\begin{aligned}&f^T(Af(y(r))+AGw(r))f(Af(y(r))+AGw(r))\\ &\quad=\|f(Af(y(r))+AGw(r))\|^2\\ &\quad\leq\|Af(y(r))+AGw(r)\|^2\\ &\quad=(Af(y(r))+AGw(r))^T(Af(y(r))+AGw(r)).\end{aligned} \tag{10}$$

Then, for two positive scalars δ_1 and δ_2, we have

$$\delta_1[x^T(r)A^TAx(r)-f^T(Ax(r))f(Ax(r))]\geq 0, \tag{11}$$

$$\begin{aligned}&\delta_2[(Af(y(r))+AGw(r))^T(Af(y(r))+AGw(r))\\ &\quad-f^T(Af(y(r))+AGw(r))f(Af(y(r))+AGw(r))]\geq 0.\end{aligned} \tag{12}$$

Using (11) and (12), we obtain a new bound for $\Delta V(x(r))$ as

$$\begin{aligned}\Delta V(x(r)) &\leq f^T(Ax(r))[P-Q]f(Ax(r))+f^T(Af(y(r))+AGw(r))Q\\ &\quad\times f(Af(y(r))+AGw(r))+f^T(Ax(r))PGw(r)\\ &\quad+w^T(r)G^TPf(Ax(r))+w^T(r)G^TPGw(r)\\ &\quad-x^T(r)Px(r)+2f^T(Ax(r))M[Ax(r)-f(Ax(r))]\\ &\quad-2f^T(y(r))M[y(r)-f(y(r))]+\delta_1[x^T(r)A^TAx(r)\\ &\quad-f^T(Ax(r))f(Ax(r))]+\delta_2[(Af(y(r))\end{aligned}$$