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On the performance of deterministic beamformers: A trade-off between array gain and attenuation

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ABSTRACT

It is customary to look over deterministic beamforming techniques as designs that offer a trade-off between mainlobe width and sidelobe level. In this work, we take into consideration that noise reduction and interference rejection are actually more useful metrics for the design of practical systems, and we present a novel analysis as a first step to understand the behavior and limitations of the deterministic beamformers from this system level perspective. The obtained results show that a trade-off between both metrics exists, and they illustrate some misconceptions about the traditionally assumed optimal designs. Finally, a method to approximately calculate the best attainable performance of any deterministic beamformer is presented.

1. Introduction

Beamforming is an array signal processing technique that provides a versatile form of spatial filtering. The existing beamforming techniques can be mainly classified into two groups [1]: *deterministic beamforming* and *data-dependent beamforming*. In the former, the designs aim to generate a fixed response for all possible scenarios, where sidelobe level and mainlobe width are typical performance metrics. In the latter, the designs depend on the statistics of the incoming data, where output signal-to-interference plus noise ratio (SINR) is a common performance metric.

Currently, the application requirements at a system level are usually present in terms of interference power and noise power at the output of the beamformer [2–4], and normally they cannot be understood simply as a single requirement on the interference-plus-noise power. These requirements can be alternatively expressed in terms of the beamformer's ability to mitigate the noise (*array gain*) and reject the interferences (*attenuation*), and they can be represented in a curve that relates both metrics. On the other hand, each beamforming technique is inherently characterized by a performance curve containing the array gain and attenuation values that it can offer, each point corresponding to a specific design. A natural concern is then to accurately quantify the performance curves, since they allow us to know which designs can be eligible for the application of interest. Fig. 1a depicts this idea. This clearly casts doubts on the optimality of some commonly used beamforming performance metrics, and it shows that array gain and attenuation may be better metrics.

Recently, the authors of [4] studied the trade-off between array gain and attenuation of some data-dependent beamformers, and they proposed a new beamformer that allows the control of this trade-off. However, a similar study about deterministic beamformers is also necessary since unfortunately most data-dependent beamformers do not allow this control and they fail in some scenarios [1,5–7]. In contrast, deterministic beamformers constitute a robust [1,8,9] and simpler option to be implemented. Moreover, they offer adequate solutions when the desired signal and the interferences are known to be confined in different spatial regions, as in GNSS reference stations [3], radio telescopes for interferometry and the over-the-horizon radar.

In this work we shed some light on the relation between attenuation and array gain of the most relevant deterministic techniques. We compare their behavior and

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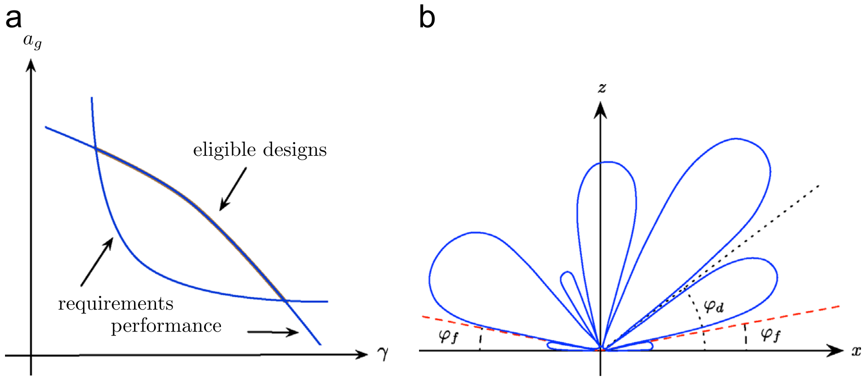

Fig. 1. (a) Example of the requirements and performance curves in terms of array gain (a_g) versus attenuation (γ). (b) Scenario of interest and example of a possible beam pattern.

limitations in a realistic scenario, and we show that the Dolph–Chebychev beamformer, which is usually adopted as the optimal solution to reject the signals coming from a given spatial sector, is not the best design from a system level perspective. In order to obtain a benchmark to evaluate the performance of any beamformer, we also present a method to approximately calculate the optimal performance curve.

2. Problem statement

Let us consider that an N-element uniform linear array receives $s(t)$, $m_1(t), \ldots, m_M(t)$ and $n(t)$, which are the baseband representations of the desired signal, M interferences and additive white noise respectively. Assuming that the array narrow-band condition is fulfilled [1], the baseband equivalent of the beamformer output signal is

$$y(t) = \mathbf{w}^H \mathbf{v}(\theta_0) s(t) + \sum_{k=1}^{M} \mathbf{w}^H \mathbf{v}(\theta_k) m_k(t) + \mathbf{w}^H \mathbf{n}(t) \tag{1}$$

where $\mathbf{w} \in \mathbb{C}^N$ contains the beamforming weights, H denotes the conjugate transpose operation, $\mathbf{v}(\theta) \in \mathbb{C}^N$ is the *steering vector* at a given direction-of-arrival (DOA) θ, and $\theta_0, \theta_1, \ldots \theta_M$ are the DOAs of the desired signal and interferences respectively, defined as the arrival angles with respect to the array axis. Finally, $\mathbf{n}(t) \in \mathbb{C}^N$ contains the received noise at each element of the array.

In the applications of interest we cannot assume that the DOAs of the interferences are known. Instead, the interferences are assumed to arrive from elevations lower than a value φ_f, and we call this region *forbidden sector*. On the other hand, the desired signal arrives from an elevation higher than a value $\varphi_d > \varphi_f$, and we call this region *desired sector*. The remaining area is the *transition sector*. The elevations belonging to the forbidden sector correspond to $\theta \in [0, \varphi_f] \cup [\pi - \varphi_f, \pi]$, and for the desired one $\theta \in [\varphi_d, \pi - \varphi_d]$.

The aim of the beamformer is to find the weights $\mathbf{w}$ that verify a particular requirements on the array response or *beam pattern* $\mathbf{w}^H \mathbf{v}(\theta)$. Fig. 1b shows a scheme of the described scenario and an example of a possible beam pattern. From all existing metrics related to $\mathbf{w}$, we are interested in the attenuation γ and the array gain a_g of the corresponding beam pattern, defined as

$$\gamma^{-1} := \max\{|\mathbf{w}^H \mathbf{v}(\theta)|^2 / |\mathbf{w}^H \mathbf{v}(\theta_0)|^2 : \theta \in [0, \varphi_f] \cup [\pi - \varphi_f, \pi]\} \tag{2}$$

$$a_g := |\mathbf{w}^H \mathbf{v}(\theta_0)|^2 / |\mathbf{w}^H \mathbf{w}| \tag{3}$$

Note that the attenuation definition is consistent with the worst-case requirements of the considered applications, and the noise definition considers the special case of spatial white noise and identical noise spectra at each sensor [1].

The goal of the paper is then to study the relation between γ and a_g of the current deterministic beamformers for linear arrays and find an optimal performance curve to obtain a benchmark that let us evaluate their performance. The inter-element spacing of the array is chosen to be half wavelength through all the paper since the corresponding beam pattern presents the best resolution without ambiguity.

3. Array gain versus attenuation trade-off

3.1. Deterministic beamforming techniques

We discuss here how to adapt the existing deterministic techniques to our scenario. The first step is to select those methods in which either a_g or γ can be modified deliberately by the designer. This is only the case of the Main Response Axis (MRA) methods [1], which assure an accurate control of the sidelobe level.

The MRA methods mainly comprise the Spectral Weighting (SW) and the Minimum Beamwidth for Specified Sidelobe Level (MBSSL) approaches, which present a well known trade-off between sidelobe level and mainlobe width or *beamwidth*. Concretely, the MBSSL methods optimize the beamwidth for a given maximum level of sidelobes, and the Dolph–Chebychev is the best known representative because it has constant level of sidelobes. Furthermore, both approaches are characterized by having non-increasing sidelobes. This leads to a methodology of design based on building a spatial filter with pass-band given by the mainlobe and stop-band given by the sidelobes. In our scenario, the pass-band is located in the desired sector and the stop-band corresponds to the forbidden sector. The mainlobe is placed in the desired