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Extrapolation of discrete bandlimited signals in linear canonical transform domain

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ABSTRACT

This paper investigates the extrapolation problem of discrete bandlimited signals in linear canonical transform domains. First, by the generalized inverse theory of matrix, two extrapolation formulae which guarantee the minimum energy least squares error estimates are proposed. Then an iterative extrapolation algorithm is proposed, and by use of the interesting properties of discrete generalized prolate spheroidal sequences, its convergence to the minimum energy least squares error estimate is proved. Finally, numerical simulation results are given to demonstrate the effectiveness of the proposed extrapolation algorithms.

1. Introduction

The extrapolation of a bandlimited signal in Fourier transform (FT) domain in terms of a known finite segment is an essential problem in signal analysis. Various algorithms have been proposed for this problem, such as algorithms based on analytic continuation, algorithms using a series expansion in terms of prolate spheroidal functions [1,2], and iterative algorithms based on successive reduction of the mean-squared error [3–6]. These algorithms have been found potential applications in FT based image processing, synthetic aperture radar, spectrum estimation and communication theory [1–6]. However, all of these algorithms were derived from the classical Fourier bandlimited signal viewpoint. In fact, there are many signals which need to be extrapolated are not bandlimited in FT domain.

Linear canonical transform (LCT), which is a generalization of FT and fractional FT, is a four-parameter linear integral transform [7,8]. Because of the extra degrees of freedom, LCT is more flexible and has been shown to be a powerful tool for quantum mechanics [9], signal processing [10–16], optics [17–21], and communications [22–24]. In the real applications of LCT, due to the finite-sized optical systems and transducer limitations [18], signals that need to be extrapolated are often bandlimited in LCT domain. Recently, several FT based bandlimited signal extrapolation algorithms have been extended to the case of LCT. In [25], Sharma and Joshi generalized the FT based Gerchberg-Papoulis (GP) algorithm to fractional bandlimited signals. In earlier papers, based on a series expansion in terms of generalized prolate spheroidal wave functions, we introduced an extrapolation formulation [26], and proposed an error energy reduction procedure based iterative extrapolation algorithm for bandlimited signals in LCT domain [27]. In [28], Shi et al. proposed a new

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extrapolation formulation of the GP algorithm for bandlimited signals in LCT domain and presented a fast convergence algorithm for the new formulation. These algorithms resolve the extrapolation problem of bandlimited signals in LCT domain, rather than in classical FT domain. However, all of these algorithms are suitable for continuous signals only. Unfortunately, there are practical difficulties of measuring continuous segment of signals in real applications. In fact, due to the widely used digital signal processors and/or systems, the known information is often a finite segment of a discrete bandlimited signal in LCT domain. Therefore, it is often required to extrapolate the unknown part of a discrete bandlimited signal in LCT domain in real applications.

In this paper, we consider the extrapolation problem for discrete bandlimited signals in LCT domain. First, by the pseudoinverse theory of matrix, an extrapolating formula which guarantees that the extrapolation result achieves the minimum energy least squares error estimate is proposed. Then the generalized inverse theory is used to set up an extrapolation algorithm which guarantees that the extrapolation result also achieves the minimum energy least squares error estimate. Finally, an iterative extrapolation algorithm is proposed, and by use of the interesting properties of discrete generalized prolate spheroidal sequences (DGPSSs), its convergence is proved. Numerical simulation results are given to demonstrate the effectiveness of the proposed extrapolation algorithms.

2. Preliminaries

2.1. Notations

Throughout this paper, superscripts $*$, T and H denote complex conjugate, transpose and Hermitian transpose, respectively. The notation $R(A)$ represents the rank of matrix A.

2.2. The LCT of discrete signals

For any discrete signal $\mathbf{f}=\cdots,f(-1),f(0),f(1),\cdots$, its LCT with parameters $\mathfrak{M}=(a,b,c,d)$, or briefly $\mathfrak{M}$–LCT is defined as [29]

$$F_{\mathfrak{M}}(u)=\sum_{n=-\infty}^{\infty} f(n)\mathcal{K}_{\mathfrak{M}}(n,u), \tag{1}$$

with

$$\mathcal{K}_{\mathfrak{M}}(n,u)=\sqrt{\frac{1}{i2\pi b}}e^{\frac{id}{2b}u^2}e^{-\frac{i}{b}un}e^{\frac{ia}{2b}n^2}. \tag{2}$$

Here a,b,c,d are real numbers satisfying $ad-bc=1$. The condition $ad-bc=1$ implies that there are only three freedoms a, b and d in the expression (2). Without loss of generality, we assume $b>0$ in this paper. $\mathbf{f}$ can be represented by its $\mathfrak{M}$–LCT as

$$f(n)=\int_{-\pi b}^{\pi b} F_{\mathfrak{M}}(u)\mathcal{K}^*_{\mathfrak{M}}(n,u)\,du,\quad n=0,\pm1,\pm2,\cdots. \tag{3}$$

It is easy to derive that

$$\sum_{n=-\infty}^{\infty}|f(n)|^2=\int_{-\pi b}^{\pi b}|F_{\mathfrak{M}}(u)|^2du=E, \tag{4}$$

where E is the total energy of $\mathbf{f}$. Throughout this paper, we restrict our attention to signals whose total energy is finite.

Let $\sigma\le\pi b$ be a positive number. If the LCT $F_{\mathfrak{M}}(u)$ of $\mathbf{f}$ vanishes for $\pi b\ge|u|>\sigma$, we say that $\mathbf{f}$ is σ bandlimited in $\mathfrak{M}$–LCT domain, or equally, we say that $\mathbf{f}$ is $\mathfrak{M}$–bandlimited to σ.

2.3. The DGPSSs

The DGPSSs are defined as the normalized solutions of the equations [29]

$$\lambda_k v_k(n)=\sum_{m=-N}^{N} G_{\mathfrak{M}}(n,m)v_k(m),\ n=0,\pm1,\cdots;\ k=0,1,\ldots,2N, \tag{5}$$

where $G_{\mathfrak{M}}(n,m)$ is given by [30]

$$G_{\mathfrak{M}}(n,m)=\frac{\sigma}{\pi b}e^{\frac{ia}{2b}(m^2-n^2)}\mathrm{sinc}[\sigma(n-m)/b] \tag{6}$$

with $\mathrm{sinc}(x)=\sin(x)/x$.

The DGPSSs $\mathbf{v}_k$ were found to be orthogonal, i.e.,

$$\sum_{n=-\infty}^{\infty} v_k(n)v_l^*(n)=\delta(k-l). \tag{7}$$

The eigenvalues λ_k were found to be positive and less than unity. We can sort them as

$$0<\lambda_{2N}<\lambda_{2N-1}<\cdots<\lambda_0<1. \tag{8}$$

Besides, the DGPSSs were also found to be $\mathfrak{M}$–bandlimited to σ, hence by the unitary property of LCT, we have

$$\sum_{n=-\infty}^{\infty} v_k(n)G^*_{\mathfrak{M}}(n,m)=\int_{-\sigma}^{\sigma}V_k(u)\mathcal{K}^*_{\mathfrak{M}}(m,u)du=v_k(m). \tag{9}$$

Here $V_k(u)$ and $\mathcal{K}_{\mathfrak{M}}(m,u)$ are the $\mathfrak{M}$–LCTs of $\{v_k(n), n=0,\pm1,\cdots\}$ and $\{G_{\mathfrak{M}}(n,m), n=0,\pm1,\cdots\}$, respectively.

For more details of DGPSSs, see [29].

3. Extrapolation of discrete bandlimited signals in LCT domain

Suppose $\mathbf{f}$ is $\mathfrak{M}$–bandlimited to σ, the given information is a set of observations $g(n)=f(n), |n|\le N$. Our problem is to determine the unknown part of $\mathbf{f}$ from the known observations $g(n)$, $|n|\le N$.

Before proceeding to discuss the extrapolation problem, we establish operator notations which we follow for the remainder of the paper. Define $\mathbf{B}^\sigma_{\mathfrak{M}}$ as the $\mathfrak{M}$–bandlimiting operator with

$$(\mathbf{B}^\sigma_{\mathfrak{M}}f)(n)=\int_{-\sigma}^{\sigma}F_{\mathfrak{M}}(u)\mathcal{K}^*_{\mathfrak{M}}(n,u)\,du. \tag{10}$$

That is, $\mathfrak{M}$–bandlimiting a signal $\mathbf{f}$ produces a signal $\mathbf{B}^\sigma_{\mathfrak{M}}\mathbf{f}$ whose $\mathfrak{M}$–LCT agrees with the $\mathfrak{M}$–LCT of $\mathbf{f}$ for $|u|\le\sigma$, and vanishes for $\pi b\ge|u|>\sigma$. Let $\mathcal{B}=\{b(n,m),-\infty\le n,m\le\infty\}$ denote an $\infty\times\infty$ matrix with the $(n,m)th$ element given by $b(n,m)=G_{\mathfrak{M}}(n,m)$. It is easy to see that $\mathcal{B}$ is idempotent and Hermitian, i.e., $\mathcal{B}^2=\mathcal{B}=\mathcal{B}^H$. By the definition (1) of LCT, the result $\mathbf{B}^\sigma_{\mathfrak{M}}\mathbf{f}$ of $\mathfrak{M}$–bandlimiting $\mathbf{f}$ can be written as