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Improved bi-equiripple variable fractional-delay filters

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ABSTRACT

This paper presents a bi-minimax method for designing an odd-order variable fractional-delay (VFD) finite-impulse-response (FIR) digital filter such that both the peak errors of its variable frequency response (VFR) and VFD response can be simultaneously suppressed. The bi-minimax design iteratively minimizes a mixed error function involving both the VFR-peak-error and VFD-peak-error subject to the second-order-cone (SOC) constraints on the VFR errors and linear-programming (LP) constraints on the VFD errors. As compared with the existing SOC-based minimax design that minimizes the VFR-peak-error only, this odd-order bi-minimax design suppresses the VFD-peak-error and flattens both the VFR errors and VFD errors simultaneously. Consequently, both the two errors are made nearly equi-ripple (bi-equiripple). An example is given for showing the simultaneous suppression of the two kinds of peak errors and verifying the effectiveness of the odd-order bi-minimax design approach.

1. Introduction

Variable fractional-delay (VFD) digital filters are useful in various signal processing applications such as sampling rate conversion [1]. So far, many efficient methods have been developed for designing both finite-impulse-response (FIR) and all-pass VFD digital filters. Most of the existing design methods obtain VFD filters by approximating the ideal variable frequency response (VFR) in the weighted-least-squares (WLS) sense or minimax sense [2–16]. The WLS design minimizes the integral-squared-error of VFR response, whereas the minimax design minimizes the VFR-peak-error (maximum absolute VFR error). Generally speaking, there is a trade-off between the two errors, i.e., one cannot minimize the two errors simultaneously. In other words, neither the WLS design nor the minimax design can minimize the total error energy and VFR-peak-error simultaneously. As far as the minimax design is concerned, minimizing the VFR-peak-error usually results in a large VFD-peak-error, which may not be acceptable in practical applications if the VFD-peak-error is too large. Therefore, it is desirable to develop a new minimax method that can suppress the VFD-peak-error while maintaining the VFR-peak-error at an acceptable level. In [11], an odd-order bi-minimax design is presented for suppressing the VFD-peak-error of an odd-order FIR VFD filter by approximating the non-linear constraints on the VFD errors as bi-linear constraints and then an iterative alternating optimization scheme is proposed for solving the non-linear bi-minimax design problem. Originally, such a bi-minimax design is a non-linear minimization problem because the constraints on the VFD errors are highly non-linear.

To solve this non-linear problem, we first linearize the non-linear constraints as linear ones and then solve the bi-minimax design. This paper generalizes the odd-order bi-minimax design approach in [11] for improving the accuracy of the VFD-peak-error suppression. The basic idea is to utilize a weighting function for the VFD error and iteratively updating the weighting function such that the VFD errors are made almost flat. As compared with [11], the non-linear constraints on the VFD errors are formulated in a different way through involving the denominator term in

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the VFD error function. By enhancing these two aspects, the VFD-peak-error can be further suppressed as compared with the primary (plain) odd-order bi-minimax design [11]. Therefore, this denominator-involved reweighted odd-order bi-minimax design can achieve a more accurate odd-order bi-minimax design than the plain one that does not use the denominator in the VFD error function and weighting function.

The key point of the bi-minimax design is to minimize a mixed error function that involves both the VFR-peak-error and the VFD-peak-error through using a scaling factor. This scaling factor can be adjusted to control the relative weights of the two peak errors and thus control the magnitudes of the two peak errors. Minimizing the mixed error function subject to the second-order-cone (SOC) constraints on the VFR errors and the linearized constraints on the VFD errors leads to an optimal bi-minimax solution. This bi-minimax problem can be efficiently solved by using the well-known software *SeDuMi* [17]. An odd-order design example is provided for illustrating the performance improvement over the existing second-order-cone (SOCP) design and the plain odd-order bi-minimax design [11] that ignores the denominator term in the VFD error expression as well as does not use a weighting function.

2. Transfer function and error expressions

The ideal frequency response of an odd-order VFD filter is given by

$$H_I(\omega, d) = e^{-j\omega d} \tag{1}$$

where $\omega \in [0, \alpha\pi]$ is the normalized angular frequency, $d \in [0, 1]$ is the VFD parameter, and the parameter $\alpha \in (0, 1]$ specifies the passband edge of interest. To exploit the coefficient symmetry, we perform the substitution

$$d = \frac{1}{2} + p, \quad p \in [-0.5, 0.5] \tag{2}$$

and change the original VFD parameter d into a new parameter p. By substituting (2) into (1), we yield

$$H_I(\omega, d) = e^{-j\omega/2} \hat{H}_I(\omega, p) \tag{3}$$

with

$$\hat{H}_I(\omega, p) = e^{-j\omega p}. \tag{4}$$

To approximate (3), we utilize the variable transfer function

$$H(z, p) = \sum_{n=-N}^{N+1} h_n(p) z^{-n} \tag{5}$$

by parameterizing the coefficients $h_n(p)$ as

$$h_n(p) = \sum_{m=0}^{M} a(n, m) p^m \tag{6}$$

where $h_n(p)$ are the M-th degree polynomials of the VFD parameter p. Thus,

$$\begin{aligned} H(z, p) &= \sum_{n=-N}^{N+1} \sum_{m=0}^{M} a(n, m) z^{-n} p^m \\ &= \sum_{m=0}^{M_e} F_m(z) p^{2m} + \sum_{m=1}^{M_o} G_m(z) p^{2m-1} \end{aligned} \tag{7}$$

where M_e, M_o are defined as

$$M_e = \left\lfloor \frac{M}{2} \right\rfloor, \quad M_o = \left\lceil \frac{M}{2} \right\rceil \tag{8}$$

$\lfloor\ \rfloor$, $\lceil\ \rceil$ are floor and ceiling functions, respectively. Also, the odd-order fixed-coefficient FIR digital filters

$$\begin{aligned} F_m(z) &= \sum_{n=-N}^{N+1} a(n, 2m) z^{-n} \\ G_m(z) &= \sum_{n=-N}^{N+1} a(n, 2m-1) z^{-n} \end{aligned} \tag{9}$$

are called sub-filters in the Farrow structure [1]. To reduce the VFD filter complexity, we can exploit the coefficient symmetry

$$a(1-n, m) = (-1)^m \cdot a(n, m) \tag{10}$$

in the odd-order VFD filter design [4]. That is, the sub-filters $F_m(z)$ have even-symmetric coefficients, and $G_m(z)$ have odd-symmetric (anti-symmetric) coefficients. Substituting (10) into (9) obtains

$$\begin{aligned} F_m(z) &= z^{-1/2} \hat{F}_m(z) \\ G_m(z) &= z^{-1/2} \hat{G}_m(z) \end{aligned}$$

with

$$\begin{aligned} \hat{F}_m(z) &= \sum_{n=1}^{N+1} a(n, 2m) [z^{(n-1/2)} + z^{-(n-1/2)}] \\ \hat{G}_m(z) &= -\sum_{n=1}^{N+1} a(n, 2m-1) [z^{(n-1/2)} - z^{-(n-1/2)}]. \end{aligned}$$

Thus, the transfer function (7) can be rewritten as

$$H(z, p) = z^{-1/2} \hat{H}(z, p) \tag{11}$$

with

$$\hat{H}(z, p) = \sum_{m=0}^{M_e} \hat{F}_m(z) p^{2m} + \sum_{m=1}^{M_o} \hat{G}_m(z) p^{2m-1}. \tag{12}$$

By comparing (3) with (11), it is clear that we only need to approximate $\hat{H}_I(\omega, p)$ in (4) by using $\hat{H}(z, p)$. Furthermore, the sub-filters $\hat{F}_m(z)$ and $\hat{G}_m(z)$ do not need to have the same orders because using different-orders may further reduce the VFD filter complexity in terms of the number of independent multipliers, which has been shown in the odd-order weighted-least-squares (WLS) design [4]. Assume that the order of $\hat{F}_m(z)$ is N_{em}, and the order of $\hat{G}_m(z)$ is N_{om}, where e denotes even-symmetric, and o denotes odd-symmetric. Then, the frequency responses of the different-orders $\hat{F}_m(z)$ and $\hat{G}_m(z)$ can be rewritten as

$$\begin{aligned} \hat{F}_m(\omega) &= \sum_{n=1}^{N_{em}+1} b_{em}(n) \cos\left(n - \frac{1}{2}\right)\omega = \mathbf{c}_m^T(\omega) \mathbf{b}_{em} \\ \hat{G}_m(\omega) &= (-j) \cdot \sum_{n=1}^{N_{om}+1} b_{om}(n) \sin\left(n - \frac{1}{2}\right)\omega = (-j) \cdot \mathbf{s}_m^T(\omega) \mathbf{b}_{om} \end{aligned} \tag{13}$$

where

$$\begin{aligned} b_{em}(n) &= 2a(n, 2m), \quad n = 1, 2, \ldots, (N_{em} + 1) \\ b_{om}(n) &= 2a(n, 2m-1), \quad n = 1, 2, \ldots, (N_{om} + 1) \end{aligned} \tag{14}$$

$$\mathbf{c}_m^T(\omega) = \left[\cos\left(\frac{\omega}{2}\right) \ \cos\left(\frac{3\omega}{2}\right) \ \cdots \ \cos\left(N_{em} + \frac{1}{2}\right)\omega \right]$$