Signal Processing 94 (2014) 138–143

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Fast communication

Improving noisy sensor positions using accurate inter-sensor range measurements

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ARTICLE INFO

Article history:
Received 3 December 2012
Received in revised form
3 May 2013
Accepted 17 June 2013
Available online 1 July 2013

Keywords:
Sensor position refinement
Range measurements
Constrained Cramèr–Rao Lower Bound (CRLB)
Constrained maximum-likelihood estimation (CMLE)

ABSTRACT

In this paper, we consider the problem of improving noisy sensor positions using accurately measured inter-sensor distances. A novel two-step algorithm is proposed. In Step-1, the multidimentional scaling (MDS) technique is applied to the range measurements and a shifted, rotated and possibly reflected version of the true sensor positions is obtained. Step-2 converts the original problem into a generalized trust region sub-problem (GTRS) that can be solved globally via simple bisection searching. The proposed algorithm is shown to be asymptotically efficient. Simulations verify the efficiency of the proposed algorithm.

1. Introduction

Source localization using an array of sensors whose positions are known is essential for applications such as navigation, teleconferencing and surveillance. The source localization accuracy, however, is very sensitive to the presence of sensor position errors [1–3]. We may suffer from great performance loss even when the known sensor positions contain a small amount of errors. The sensor position errors are common in practical source localization systems. It can come from the sensor drifting due to e.g., sea waves or the estimation error when the sensor positions are found via sensor network self-localization.

Measuring the sensor positions directly to reduce the sensor position errors may not always be feasible, especially when the sensors are deployed remotely (such as in outer space) or in a hostile environment. Instead, the commonly used approaches are to first obtain measurements that are dependent on the true sensor positions and then utilize them to refine the known sensor positions. Among them, an effective technique is to use calibration emitters at perfectly known [4,5], imprecisely known [6] or completely unknown positions [1,7–10], where the sensors measure the signals from calibration emitters to refine their positions. This paper considers a different approach that exploits inter-sensor range measurements. It has the advantage that the deployment of additional calibration emitters is no longer needed.

The time-based approach is perhaps the most widely used ranging method [11]. In satellite-based electronic reconnaissance systems, laser ranging can measure inter-satellite distances to within a few millimeters through determining the round-trip time for laser pulses to return to the sender. In sensor networks, the inter-sensor distance between two synchronized sensors can be found by exchanging time-stamped data packets. Specifically, the receiving sensor can extract the time stamp the

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[1] Le Yang's work is supported by the Startup Fund and the Youth Foundation of Jiangnan University (Contract no. JUSRP11234).

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http://dx.doi.org/10.1016/j.sigpro.2013.06.027

transmitting sensor includes in the data packet and subtract it from the time when the packet is received to deduce the inter-sensor distance. Another example is the use of the ultra-wide band (UWB) ranging in sensor networks (see e.g., [11]). Owing to the ability of UWB pulses in resolving multipath and penetrating obstacles [12,13], a ranging precision of a few centimeters can be achieved under small noise condition.

We shall consider in this work the scenario where the noise in the measured inter-sensor distances is negligible compared with the sensor position errors. For instance, in satellite-based electronic reconnaissance systems, the satellite position uncertainty is on the order of kilometers while the laser ranging has a precision of a few millimeters. As another example, in UWB sensor networks, the ranging error of a few centimeters may still be much less than the sensor position error that could be in meters, due to e.g., sensor drifting. We shall propose a novel method that can efficiently exploit the accurate inter-sensor distance measurements to refine the known but erroneous sensor positions. In the algorithm development, the obtained inter-sensor distances will be considered *noiseless*.

The newly developed algorithm is a maximum likelihood (ML) estimator for the true sensor positions, which is subject to equality constraints imposed by the precise inter-sensor range measurements and is a non-convex optimization problem. The proposed algorithm has two processing steps and it guarantees to find the globally optimal solution to the ML estimation problem. Its Step-1 processing applies the multidimensional scaling (MDS) technique to the inter-sensor range measurements to obtain estimates of the true sensor positions subject to unknown shift, rotation and reflection. The Step-2 explores the Step-1 output to transforms the non-convex ML estimation problem into a generalized trust region subproblem (GTRS) and solves it globally using the simple bisection searching. As an ML estimator, the two-step algorithm is asymptotically efficient. That is, it can attain the Cramèr–Rao Lower Bound (CRLB) for the problem of sensor position refinement using noise-free inter-sensor distances. Simulations verify the efficiency of the proposed algorithm and illustrate the impact of the neglected inter-sensor range measurement noise on its performance. We find that the performance degradation is marginal when the sensor position error is large. This corroborates the validity of the algorithm development.

It is worthwhile to point out that in literatures, the problem of locating the sensors via range measurements has been extensively examined (see e.g., [14–16] and references therein). Most of them utilize noisy range measurements and identify the sensor positions under the framework of unconstrained optimization, where the inter-sensor distances are soft constraints that the estimated sensor positions only need to satisfy approximately. However, in our work, we consider the use of accurate inter-sensor distances and they become equality constraints that the refined sensor positions satisfy exactly. As a result, the sensor position refinement is achieved via solving an equality-constrained optimization problem, which cannot be directly tackled by many methods available in literatures on range-based sensor node localization. Other related algorithms include the linear minimum mean square error (LMMSE) estimators for improving the known sensor positions using the signal measurements from calibration emitters [5,6,9]. They can be modified to process accurate inter-sensor distances. Nevertheless, due to the use of first-order Taylor-series expansion, the refined sensor positions may not always satisfy the constraints imposed by the precisely known inter-sensor distances and the LMMSE estimator would be inferior to the proposed algorithm when the sensor position error large. This will be illustrated in the simulation section.

2. Problem formulation

Consider an array of M sensors on a 2-D plane. The known but noisy position of sensor i, $i=1,2,\ldots,M$, is denoted by $\mathbf{s}_i=\mathbf{s}_i^o+\mathbf{n}_i$, where $\mathbf{s}_i^o=[x_i^o,y_i^o]^T$ represents the unknown true sensor position and $\mathbf{n}_i$ is the position error in $\mathbf{s}_i$. Collecting $\mathbf{s}_i$ yields a $2M\times 1$ sensor position vector $\mathbf{s}=\mathbf{s}^o+\mathbf{n}$, where $\mathbf{s}^o=[\mathbf{s}_1^{oT},\mathbf{s}_2^{oT},\ldots,\mathbf{s}_M^{oT}]^T$ and $\mathbf{n}=[\mathbf{n}_1^T,\mathbf{n}_2^T,\ldots\mathbf{n}_M^T]^T$. We model $\mathbf{n}$ as a Gaussian random vector with zero mean and covariance matrix $\mathbf{Q}_s$, as in [8,2,5,6,9,10].

To improve the sensor position information, inter-sensor distances are accurately measured. Let $r_{i,j}$ be the range measurement between sensor pair i and j such that

$$r_{i,j}=\|\mathbf{s}_i^o-\mathbf{s}_j^o\| \tag{1}$$

where $i,j=1,2,\ldots,M$, $i<j$ and $\|*\|$ represents the Euclidean norm. We shall consider the fully connected scenario where there exists an accurate range measurement for every sensor pair. Collecting the available range measurements yields an $M(M-1)/2\times 1$ vector

$$\mathbf{r}=[r_{1,2},r_{1,3},\ldots,r_{1,M},r_{2,3},\ldots,r_{M-1,M}]^T. \tag{2}$$

The objective is to reduce the noise in the known sensor positions $\mathbf{s}_i$ as much as possible through exploring the accurate range measurements in $\mathbf{r}$.

3. Proposed algorithm

To improve the noisy sensor positions, we estimate the true sensor position vector $\mathbf{s}^o$ via solving

$$\min_{\mathbf{s}^o}\ (\mathbf{s}-\mathbf{s}^o)^T\mathbf{Q}_s^{-1}(\mathbf{s}-\mathbf{s}^o)$$
$$\text{s.t. } \|\mathbf{s}_i^o-\mathbf{s}_j^o\|=r_{i,j},\ i,j=1,2,\ldots,M,\ i<j. \tag{3}$$

The cost function comes from that the known sensor position vector $\mathbf{s}$ is Gaussian distributed and an unbiased estimate of $\mathbf{s}^o$. The constraints in (3) are imposed by the accurately measured inter-sensor distances $r_{i,j}$. Besides, the estimator in (3) is termed as the constrained maximum-likelihood estimator (CMLE) in [17]. It has been shown [17,18] that the globally optimal solution to (3) is asymptotically efficient, i.e, the CMLE can reach the constrained CRLB of $\mathbf{s}^o$, which is given in (15) in Appendix A.

Finding the globally optimal solution to (3) is non-trivial. This is mainly due to the non-convexity of the feasible region formed by all the possible values of $\mathbf{s}^o$ that satisfy the equality constraints. Commonly used numerical methods,