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Fast communication

A sparse recovery algorithm for DOA estimation using weighted subspace fitting ☆

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ABSTRACT

A new algorithm involving sparse recovery is proposed to address the problem of direction-of-arrival (DOA) estimation using weighted subspace fitting (WSF). The proposed algorithm proves to be a modified version of ℓ_1-SVD by using an optimal weighting matrix, wherein a scheme of regularization between sparsity penalty and subspace fitting error is also given for all SNR range. Numerical simulations verify the efficiency of the proposed algorithm and illustrate the performance improvement in low SNR.

1. Introduction

Direction of arrival (DOA) estimation has been an important topic during the last decades [1] due to its wide application in radar, sonar, radio astronomy, etc. The traditional way to solve this problem includes the maximum likelihood (ML) estimators [2] and subspace-based approaches [3]. Recently, the techniques of sparse recovery have provided a new perspective of DOA estimation by exploiting the spatial sparsity in the array signal model [4–6,8–9], and the super-resolution property and ability of resolving coherent sources have attracted a lot of attention. An early work of applying sparse recovery into DOA estimation is the global matched filter (GMF) [4] which exploited the beamformer samples for DOA estimation based on uniform circular array. The most successful one is ℓ_1-SVD [5] which employs ℓ_1-norm to enforce sparsity and singular value decomposition to reduce complexity and sensitivity to noise. The optimization problem in ℓ_1-SVD can be deemed as a subspace fitting [7] procedure, whereas the optimality of the weighting matrix is not preserved. Another problem is that the scheme of regularization between sparsity penalty and subspace fitting error is suboptimal in low signal-to-noise ratio (SNR). Some recently proposed methods including sparse iterative covariance-based estimation (SPICE) [8] and sparse representation of array covariance vectors (SRACV) [9] also employed ℓ_1-norm penalty, while they addressed the problem in the correlation domain instead of the data domain. Actually the rigorous constraint to enforce sparsity should be ℓ_1-norm instead of ℓ_1-norm, whereas the optimization problem involving ℓ_0-norm is NP-hard. An alternative strategy named joint $\ell_{2,0}$ approximation (JLZA) was given in Ref. [10], where a class of Gaussian functions was used to approximate the ℓ_0-norm constraint; however, it is difficult to choose the appropriate parameters for all scenarios in real applications.

In this communication, sparse recovery is introduced to solve the problem of weighted subspace fitting (WSF)

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[11] whose solution was shown to asymptotically attain the stochastic Cramér–Rao lower bound (CRLB) [12]. In the proposed algorithm, which is termed as sparse recovery for weighted subspace fitting (SRWSF), a regularization scheme based on the optimal weighting matrix is given for all SNR range. Simulation results show that SRWSF efficiently improves the performance in low SNR compared with ℓ_1-SVD, besides no degradation in high SNR.

Notation: Upper (lower) bold face letters are used to denote matrices (column vectors). $\{\cdot\}^T$, $\{\cdot\}^H$, $\|\cdot\|_F$, $\|\cdot\|_p$, δ_{ik}, $diag\{\cdot\}$, $E\{\cdot\}$ and $\mathbf{I}_m$ denote transpose, conjugate transpose, Frobenius norm, ℓ_p-norm, Kronecker symbol, diagonalization, expectation, and $m\times m$ identity matrix, respectively. j is reserved for the imaginary unit $\sqrt{-1}$.

2. Array model

Assume that K narrowband far-field signals from distinct directions $\theta_1,\theta_2,\ldots,\theta_K$ impinge on a sensor array constituted by M omnidirectional elements. For simplicity, only uniform linear array (ULA) is considered, and it should be kept in mind that the proposed algorithm is suitable for arbitrary array geometry. The array output data vector for the tth snapshot can be written as

$$\mathbf{x}(t)=\mathbf{A}\mathbf{s}(t)+\mathbf{n}(t),\quad t=1,2,\ldots,L, \tag{1}$$

where $\mathbf{A}$ is the array manifold matrix, $\mathbf{s}(t)$ is the incident signal vector which is modeled as white Gaussian process, $\mathbf{n}(t)$ is the vector of sensor noise which is spatially and temporally white Gaussian with equal variance and is uncorrelated with $\mathbf{s}(t)$, and L is the total number of snapshots. The array manifold matrix $\mathbf{A}$ consists of K steering vectors

$$\mathbf{A}=[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\ldots,\mathbf{a}(\theta_K)], \tag{2}$$

where $\mathbf{a}(\theta)$ is given by

$$\mathbf{a}(\theta)=[1,v(\theta),\ldots,v^{M-1}(\theta)]^T, \tag{3}$$

and directional factor is defined as $v(\theta)=\exp(-(j2\pi d\sin\theta)/\lambda)$, where d is the interelement spacing which is set to be half a wavelength λ to avoid aliasing. In this communication, 0° denotes the broadside direction while $\pm 90^\circ$ denote the end-fire directions.

The covariance matrix of $\mathbf{x}(t)$ can be represented as

$$\mathbf{R}_x=E\{\mathbf{x}(t)\mathbf{x}^H(t)\}=\mathbf{A}\mathbf{R}_s\mathbf{A}^H+\sigma_n^2\mathbf{I}_M, \tag{4}$$

where $\mathbf{R}_s$ is the covariance matrix of incident signals, and σ_n^2 is the variance of noise. The rank of $\mathbf{R}_s$ is denoted as K', and rests with the correlation between incident signals. In the scenario of multipath propagation when coherent signals are present, K' is less than K, otherwise K' is equal to K. The eigen decomposition of $\mathbf{R}_x$ is given by

$$\mathbf{R}_x=\sum_{i=1}^{M}\mu_i\mathbf{e}_i\mathbf{e}_i^H=\mathbf{E}_s\boldsymbol{\Lambda}\mathbf{E}_s^H+\mathbf{E}_n\boldsymbol{\Gamma}\mathbf{E}_n^H, \tag{5}$$

where the eigen values $\mu_1\ge\mu_2\ge\ldots\ge\mu_{K'}>\mu_{K'+1}=\ldots=\mu_M$, $\boldsymbol{\Lambda}=diag\{\mu_1,\mu_2,\ldots\mu_{K'}\}$, $\boldsymbol{\Gamma}=diag\{\mu_{K'+1},\mu_{K'+2},\ldots,\mu_M\}$, and $\mathbf{E}_s$ and $\mathbf{E}_n$ are the corresponding eigen vector matrices which are defined as signal subspace matrix and noise subspace matrix, respectively. The matrix $\mathbf{E}_s$ stays in the same subspace of $\mathbf{A}$, i.e., it can be expressed as

$$\mathbf{E}_s=\mathbf{A}\mathbf{B}, \tag{6}$$

where $\mathbf{B}$ is a $K\times K'$ matrix having full column rank. The traditional subspace-based methods utilize the orthogonality between signal subspace and noise subspace to derive the DOA estimates.

In reality, $\mathbf{R}_x$ can only be estimated by using finite length of snapshots, and is given by

$$\hat{\mathbf{R}}_x=\frac{1}{L}\sum_{t=1}^{L}\mathbf{x}(t)\mathbf{x}^H(t). \tag{7}$$

Similarly, the eigen decomposition of $\hat{\mathbf{R}}_x$ can be written as

$$\hat{\mathbf{R}}_x=\sum_{i=1}^{M}\hat{\mu}_i\hat{\mathbf{e}}_i\hat{\mathbf{e}}_i^H=\hat{\mathbf{E}}_s\hat{\boldsymbol{\Lambda}}\hat{\mathbf{E}}_s^H+\hat{\mathbf{E}}_n\hat{\boldsymbol{\Gamma}}\hat{\mathbf{E}}_n^H. \tag{8}$$

Here L is assumed to be larger than K', otherwise the rank of $\hat{\mathbf{R}}_x$ is rather L than K'.

3. Sparse recovery for WSF

3.1. Subspace fitting with optimal weighting matrix

In Ref. [7], it was revealed that most of the prevalent DOA estimation methods can be classified into variations of the same subspace fitting problem, which can be expressed as

$$\hat{\boldsymbol{\theta}}_{SF}=\arg\min_{\boldsymbol{\theta}}\|\hat{\mathbf{E}}_s\hat{\mathbf{W}}^{1/2}-\mathbf{A}(\boldsymbol{\theta})\mathbf{P}\|_F^2, \tag{9}$$

where $\hat{\mathbf{W}}$ is the weighting matrix which varies with different methods, $\mathbf{A}(\boldsymbol{\theta})$ is the array manifold matrix parameterized by $\boldsymbol{\theta}$, and $\mathbf{P}$ is a $K\times K'$ matrix of full column rank. Note that $\hat{\mathbf{W}}=\hat{\boldsymbol{\Lambda}}$ corresponds to the subspace fitting style applied in ℓ_1-SVD, while it is not the optimal choice for the problem formulated in Eq. (9). It was proved in Ref. [7, theorem 3] that the optimal weighting matrix which gives the lowest asymptotical variance is $\hat{\mathbf{W}}_{opt}=(\hat{\boldsymbol{\Lambda}}-\hat{\sigma}_n^2\mathbf{I}_{K'})^2\hat{\boldsymbol{\Lambda}}^{-1}$, where the noise variance estimate $\hat{\sigma}_n^2$ can be obtained by averaging the $M-K'$ smallest eigen values. As the provement of optimality of $\hat{\mathbf{W}}_{opt}$ was already given in Ref. [7] and also due to page limitation, detailed discussion of this issue is omitted here. Note that the subspace fitting problem employing $\hat{\mathbf{W}}_{opt}$ is also referred to as WSF problem in Ref. [11].

Substituting the least square solution of $\mathbf{P}$ back into (9), the WSF optimization problem becomes

$$\hat{\boldsymbol{\theta}}_{WSF}=\arg\min_{\boldsymbol{\theta}} tr\{\mathbf{P}_{\mathbf{A}(\boldsymbol{\theta})}^{\perp}\hat{\mathbf{E}}_s\hat{\mathbf{W}}_{opt}\hat{\mathbf{E}}_s^H\}=\arg\min_{\boldsymbol{\theta}} Q(\boldsymbol{\theta}), \tag{10}$$

where $\mathbf{P}_{\mathbf{A}(\boldsymbol{\theta})}^{\perp}$ is the orthogonal projection matrix of $\mathbf{A}(\boldsymbol{\theta})$. It was also proved in Ref. [11] that $\hat{\boldsymbol{\theta}}_{WSF}$ is asymptotically a local ML DOA estimate based on observations of $\mathbf{C}^H(\boldsymbol{\theta})\hat{\mathbf{E}}_s$, where $\mathbf{P}_{\mathbf{A}(\boldsymbol{\theta})}^{\perp}=\mathbf{C}(\boldsymbol{\theta})\mathbf{C}^H(\boldsymbol{\theta})$ and $\mathbf{C}^H(\boldsymbol{\theta})\mathbf{C}(\boldsymbol{\theta})=\mathbf{I}_{M-K}$. The modified variable projection (MVP) method addresses the problem illustrated by Eq. (9), and gives an ultimate convergence to the true DOAs by employing a fine initialization.