Signal Processing 91 (2011) 1527–1534

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Filter design for the detection/estimation of the modulus of a vector Application to polarization data

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ARTICLE INFO

Article history:
Received 1 December 2009
Received in revised form
22 September 2010
Accepted 21 December 2010
Available online 7 January 2011

Keywords:
Filters
Image processing
Matched filters
Object detection
Polarization
Astronomy

ABSTRACT

We consider a set of M images, whose pixel intensities at a common point can be treated as the components of a M-dimensional vector. We are interested in the estimation of the modulus of such a vector associated to a compact source. For instance, the detection/estimation of the polarized signal of compact sources immersed in a noisy background is relevant in some fields like astrophysics. We develop two different techniques, one based on the maximum likelihood estimator (MLE) applied to the modulus distribution, the modulus filter (ModF) and other based on prefiltering the components before fusion, the filtered fusion (FF), to deal with this problem. We present both methods in the general case of M images and apply them to the particular case of three images (linear plus circular polarization). Numerical simulations have been performed to test these filters considering polarized compact sources immersed in stationary noise. The FF performs better than the ModF in terms of errors in the estimated amplitude and position of the source, especially in the low signal-to-noise case. We also compare both methods with the direct application of a matched filter (MF) on the polarization data. This last technique is clearly outperformed by the new methods.

1. Introduction

The detection and estimation of the intensity of compact objects – i.e. signals with a compact support either in time or space domains – embedded in a background plus instrumental noise is a problem of interest in many different areas of science and engineering. A classic example is the detection of point-like extragalactic objects such as galaxies and galaxy clusters in sub-millimetric astronomy. Regarding this particular field of interest, different techniques have proven useful in the literature. Some of the proposed techniques are frequentist, such as the standard matched filter [1], the matched multifilter [2] or the recently developed matched matrix filters [3]. Other frequentist techniques include continuous wavelets like the standard Mexican Hat [4] and other members of its family [5] and, more generally, filters based on the Neyman–Pearson approach using the distribution of maxima [6]. All these filters have been applied to real data of the cosmic microwave background (CMB), like those obtained by the WMAP satellite [7] and CMB simulated data [8] for the experiment on board the *Planck* satellite [9], which has recently started operations. Besides, Bayesian methods have also been recently developed [10]. Although we have chosen the particular case of sub-millimetric astronomy as a means to illustrate the problem of compact source detection, the methods listed above are totally general and can be used to any analogous image processing problem.

In most cases one is interested only in the intensity of the compact sources. In other cases, however, there are other properties of the signal that may be of interest. Such is the case, for example, of sources that emit electromagnetic

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doi:10.1016/j.sigpro.2010.12.008

radiation that is at least partially polarized. Polarization of light is conventionally described in terms of the Stokes parameters Q, U and V. Let us consider a monochromatic, plane electromagnetic wave propagating in the z-direction. The components of the wave's electric field vector at a given point in the space can be written as

$$E_x = a_x(t)\cos[\omega_0 t - \theta_x(t)]$$

$$E_y = a_y(t)\cos[\omega_0 t - \theta_y(t)]. \tag{1}$$

If some correlation exists between the two components in the previous equation, then the wave is *polarized*. The Stokes parameters are defined as the time averages

$$I \equiv \langle a_x^2 \rangle + \langle a_y^2 \rangle,$$

$$Q \equiv \langle a_x^2 \rangle - \langle a_y^2 \rangle,$$

$$U \equiv \langle 2a_x a_y \cos(\theta_x - \theta_y) \rangle,$$

$$V \equiv \langle 2a_x a_y \sin(\theta_x - \theta_y) \rangle. \tag{2}$$

The parameter I gives the intensity of the radiation which is always positive, while the other three parameters define the polarization state of the wave and can have either sign. Q and U are the linear polarization parameters and V indicates the circular polarization of the wave. Unpolarized radiation is described by $Q=U=V=0$.

While the total intensity of the wave is independent of the orientation of the x and y axes, the values of the other Stokes parameters are not invariant with respect to changes of the orientation of the receivers. On the other hand, the *total polarization*, defined as

$$P \equiv \sqrt{Q^2 + U^2 + V^2}, \tag{3}$$

is invariant with respect to the relative orientation of the receivers and the direction of the incoming light, and therefore it is a quantity with a clear physical meaning. For the case of purely linear polarization, $V=0$ and the previous expression reduces to $P \equiv (Q^2 + U^2)^{1/2}$. Note that in order to get P from its components Q, U, and V it is necessary to perform a non-linear operation.

Although strictly speaking Q, U and V are the components of a tensor, the invariant combination (3) gives a quantity that can be seen as the modulus of a vector. In this paper we will introduce a methodology that can be applied to any problem in which a set of images contain signals whose individual intensities can be considered as the components of a vector, but where the quantity of interest is the modulus of such a vector. For illustrative purposes, throughout the paper we will use as an example the case of light polarization, but the methods we will introduce are not limited to the example. Another possible application could be the determination of the modulus of a complex-valued signal. For example, in [11] the case of the estimation of the modulus of a complex-valued quantity whose components follow a Gaussian distribution was addressed. The techniques presented in their paper are related to our methods, but restricted to the two-dimensional case. In [12], four methods: MLE estimator, median estimator, mean estimator and Wardle and Kronberg's estimator [13] are applied to the estimation of polarization (two-dimensional case). However, these methods are applied in the cited paper to a single data and cannot be, except for the MLE, generalized to the detection of a signal with a given profile in a pixelized image, as considered in our paper. This is due to the fact that these methods lead to a system of possibly incompatible equations.

Going back to the case of point-like extragalactic objects in sub-millimetric astronomy, the polarization of the sources plays an important role in the cosmological tests derived from CMB observations. Standard cosmological models predict that CMB radiation is linearly polarized. However, some cosmological models predict in addition a possible circular polarization of CMB radiation [14]. In order to better constrain the cosmological model with observations, it is crucial to determine the degree of polarization of not only the CMB radiation but also the other astrophysical sources whose signals are mixed with it. For an excellent review on CMB polarization, see [15]. We have treated the application to the detection of linearly polarized sources in CMB maps elsewhere [16], but it is known that extragalactic radio sources can also indeed show circular polarization [17]. Besides, circular polarization occurs in many other astrophysical areas, from Solar Physics [18] to interstellar medium [19], just to put a few examples. Therefore, in this paper we aim to address the general case of linear plus circular polarization of compact sources.

In the general case we have three images Q, U and V. Different approaches can be used to deal with detection/estimation of point-like sources embedded in a noisy background. On the one hand, one can try to get the source polarization amplitude A directly on the P-map. In this approach, we will consider one filter, obtained through the MLE applied to the modulus distribution (ModF). On the other hand, we can operate with three matched filters, each one on Q, U and V followed by a quadratic fusion and square root (FF). We are trying to compare the performance of the two techniques for estimating the position and polarization amplitude of a compact source. Of course, in the case we have only the map of the modulus of a vector and the components are unknown, the FF cannot be applied but we can still use the ModF. Finally, we also apply a matched filter (MF) directly on the P-map and compare this simple method with the two new methods introduced in this paper.

In Section 2 we will develop the methodology for the case of M images, because of the possible interesting applications to the general M-dimensional case and in particular to the three-dimensional case (polarization). In Section 3 we will show the results when applying these techniques to numerical simulations of images of Q, U and V that are relevant for the detection of compact polarized sources in astrophysics. Finally, in Section 4 we give the main conclusions.

2. Methodology

2.1. The case of M images

To develop our methodology, let us consider M images with intensities $d_j(\vec{x})$ at each point $\vec{x}$ of their common domain, $j = 1, \ldots, M$. Hidden in these images there is an unknown number of signals. In this work we consider