Signal Processing 89 (2009) 963–973

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Multi-signal extension of adaptive frequency tracking algorithms

Y. Prudat *,[1], J.-M. Vesin

Signal Processing Laboratory, Federal Institute of Technology, Lausanne, Switzerland

A R T I C L E I N F O

Article history:
Received 26 June 2008
Received in revised form
10 October 2008
Accepted 3 November 2008
Available online 7 November 2008

Keywords:
Frequency tracking
Adaptive notch filter
Adaptive bandpass filter
Multivariate signals
Baroreflex

ABSTRACT

Adaptive tracking of sinusoidal signal components with time-varying amplitudes and frequencies presents a great interest in many engineering applications. In many cases, the frequency components of interest are present in more than one signal. So we propose, in this paper, a simple but efficient approach to extend existing algorithms to track frequency components simultaneously in several signals. Computer simulations and experiments on real signals demonstrate the potential of this approach in terms of estimation variance, convergence speed, and the capability to extract a frequency component common to several signals.

1. Introduction

Adaptive tracking of noisy sinusoidal signal components with time-varying amplitudes and frequencies presents a great interest in many engineering applications such as communications [1], biomedical engineering [2], and speech processing [3]. Over the years, several dedicated algorithms have been proposed in the literature. Some rely on a Kalman- [4] or an RLS-based [5] prediction algorithm, but most of them are based on an adaptive notch filter (ANF) or bandpass filter (BPF) structure. In the latter schemes, some criterion on the output of the adaptive filter is used to update the filter parameters in order to provide the tracking feedback. Ref. [6] is one of the earliest works in which an adaptive BPF for line enhancement and frequency tracking was proposed. In [7], a second-order adaptive filter for frequency estimation with fast convergence speed was described. In [8], a bank of adaptive BPFs was used for formant separation and tracking in speech signals. Ref. [9] compares four different frequency trackers, including multiple frequency tracking algorithms.

There are many domains in which a (possibly time-varying) quasi-periodic component appears in more than one signal. This is for instance the case in electroencephalograph signals, with rhythms common to several electrode signals [10,11], or in the atrial activity in surface electrocardiograms (ECGs) during atrial fibrillation, where oscillating components may be present in different lead signals [12]. One may also cite climate variability studies such as [13], in which the common frequency components between atmospheric jet-shifting mode and ocean kinetic energy are investigated. Accelerometer-based tests of mechanical systems typically aim at identifying vibration modes common to the various recording sites [14]. Sensor fusion applications, such as [15], that proposes a human activity recognition scheme based on both accelerometer and microphone signals, also correspond to this situation.

Usually, the frequency components are extracted or tracked in each signal separately and thus, the obtained results can be perturbed by artifacts or noise relative to each signal. Moreover, in some circumstances, it is of interest to track the frequency component that is indeed common to several signals and separate tracking on each

---

* Corresponding author. Tel.: +41 21 693 96 33.
E-mail address: yann.prudat@epfl.ch (Y. Prudat).

[1] Supported by the Swiss National Science Foundation (no. 205321-112136).

0165-1684/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.sigpro.2008.11.002

signal does not guarantee that the same frequency is tracked. A frequency tracking algorithm using simultaneously several signals presents two non-conflicting positive characteristics:

- Improved tracking performance in terms of convergence speed and frequency estimation variance.
- Extraction of possibly low amplitude common frequency components.

In this paper, we propose an approach to extend frequency tracking algorithms to multiple signals. Each signal is filtered by the same BPF. The central frequency is updated using a weighted sum of the update terms computed separately for each signal, with weights based on a recursive measure of the estimation accuracy in each signal.

We implement this approach for two recently proposed algorithms, based on an ANF [16] and an adaptive BPF [17] structure. The complexity induced by the weight calculation in the frequency update rules does not permit a theoretical analysis of the performance of the extended algorithms. Thus, bias, variance and convergence rate of these extensions are presented here, using Monte-Carlo simulations. We also illustrate the advantages of the proposed method in time-varying signal-to-noise ratio (SNR) environments and in the presence of correlated noise with synthetic signals and in a biomedical application.

The structure of this paper is as follows. First, the basic versions of the algorithms we extend are briefly presented. Next, we introduce our approach for multi-signal extension of these algorithms. Then, the convergence properties of the extended algorithms are studied through Monte-Carlo simulations. Finally, some examples of application to synthetic and biomedical signals are given. A short discussion concludes this paper.

2. Basic algorithms

2.1. Constrained adaptive FIR notch filter

The first algorithm extended in this paper is the constrained adaptive FIR notch filter (AFNF) proposed by Punchalard et al. in [16].

The input signal, $u(\cdot)$, is assumed to be

$$u(n) = d(n) + b(n), \tag{1}$$

where $d(\cdot)$ is a sinusoid at frequency ω_0 and $b(\cdot)$ is an additive, i.i.d. noise.

The algorithm is based on an FIR notch filter with transfer function

$$H(z,n) = 1 - 2z^{-1}\cos\omega(n) + z^{-2}, \tag{2}$$

where $\omega(n)$ is the tracking frequency.

The output of the filter is computed as

$$x(n) = u(n) - 2u(n-1)\cos\omega(n) + u(n-2), \tag{3}$$

and the conventional gradient-based update algorithm is derived as

$$\omega(n+1) = \omega(n) - 2\mu x(n)u(n-1)\sin\omega(n), \tag{4}$$

where μ is the step-size.

However, the algorithm described by Eq. (4) is biased. A bias removal technique is used to obtain the unbiased version

$$\omega(n+1) = \omega(n) - 2\mu x(n)u(n-1)\sin\omega(n) - 2\mu u(n)x(n)\sin 2\omega(n). \tag{5}$$

2.2. Oscillator based ANF (OSC-MSE)

The second algorithm extended in this paper is the OSC-MSE (oscillator based mean square error) BPF-based algorithm proposed by Liao in [17]. Its structure is shown in Fig. 1.

For an input signal $u(\cdot)$ as described in (1), the sinusoidal component $d(\cdot)$ should satisfy the oscillator equation

$$\begin{aligned} d(n) &= 2d(n-1)\cos\omega_0 - d(n-2) \\ &\equiv 2\alpha_0 d(n-1) - d(n-2). \end{aligned} \tag{6}$$

The adaptive coefficient $\alpha(\cdot)$, which tracks $\alpha_0 = \cos\omega_0$, determines the central frequency of the BPF. Its transfer function is defined as

$$H(z;n) = \frac{1-\beta}{2}\frac{1-z^{-2}}{1-\alpha(n)[1+\beta]z^{-1}+\beta z^{-2}}, \tag{7}$$

where $0<\beta<1$ controls the bandwidth of the BPF.

The reference signal $y(\cdot)$ used in the adaptive mechanism is the filter output defined by the following difference equation:

$$\begin{aligned} y(n) &= (1+\beta)\alpha(n)y(n-1) - \beta y(n-2) \\ &\quad + \frac{1-\beta}{2}(u(n) - u(n-2)). \end{aligned} \tag{8}$$

The filter defined by (7) has a zero phase shift and a unitary gain at the central frequency $\alpha(\cdot)$, so the reference signal $y(\cdot)$ is the component of $u(\cdot)$ at frequency $\alpha(\cdot)$.

In short, the adaptive algorithm is driven by a line-enhanced version of the input signal. In the OSC-MSE algorithm, the goal is to determine the value for $\alpha(n+1)$ that satisfies the discrete oscillator model. This is done by minimizing the following cost function, based on the

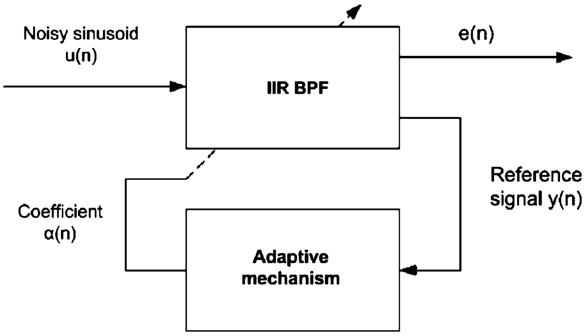

Fig. 1. Structure of the OSC-MSE algorithm.