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Linear optimal filtering for discrete-time systems with random jump delays

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ABSTRACT

This paper is concerned with the dynamic Markov jump filters for discrete-time system with random delays in the observations. It is assumed that the delay process is modeled as a finite state Markov chain. To overcome the difficulty of estimation caused by the random delays, the single random delayed measurement system is firstly rewritten as the multiplicative noise constant-delay system. Then, by applying the measurement reorganization approach, the system is further transformed into the delay-free one with Markov jump parameters. Finally, the estimator is derived by using the standard Markov jump filter theories. It is interesting to show that the presented filter for the system with random jump delays can be designed by performing two sets of standard Riccati equations with the same dimension as that of the original system. A simulation example is given to illustrate the effectiveness of the proposed result.

1. Introduction

In the signal estimation problems, it is usually assumed that the observation packets are received either immediately or with a constant and known delay [1–5]. However, in certain newly arisen engineering applications, such as underwater acoustic or mobile communications, exploration seismology, and remote control of a large number of mobile units, the observations are transmitted to the estimator via communication channels with considerable, irregular, and a priori unknown delays. Therefore, the better way to model the delay is to interpret it as a stochastic process, including its statistical properties in the system model.

In this sense, many recent works use the stochastic time delay models to treat the estimation problems or control applications [6–9]. There are several methods of achieving the state estimator for system with random observation delays, which differ in their estimation criteria and means [10–15]. In [10], the state estimation for a model involving randomly bounded sensor delays was reformulated as a stochastic parameter estimation problem. The one-step sensor delay was described as a binary white noise sequence, and a reduced-order linear unbiased estimator was designed via state augmentation. In [11,12], linear and second-order polynomial estimation algorithms from randomly delayed observations have been derived by using the covariance functions of the processes involved in the observation equation. The delay was characterized by a set of Bernoulli variables with the value of zero or one. Recently, a recursive minimum variance state estimator was proposed in [13] for linear discrete-time partially observed systems perturbed by white noises. The observations were transmitted via communication channels with random transmission times and various measurement signals would incur independent delays. Evans and Krishnamurthy [14] considered the state estimation problems for a discrete-time hidden Markov model (HMM) when the observations were delayed by a random time. The delay process was modeled as a finite state Markov chain that allowed an

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augmented state HMM to model the overall system. More recently, the H_∞ filter of discrete-time linear system with random but bounded communication delays in the output was studied in [15]. The random delay was also modeled as a finite Markov chain, and the proposed design was based on the solution of an iterative procedure of a linear matrix inequality minimum problem. In the aforementioned papers, state augmentation or LMI method has been employed. Certainly, the existing methods provide efficient and practical tools to deal with the filtering problems involving random delays. However, there are still existing some deficiencies to be improved, for example, reducing the computation complexity of state augmentation or relaxing the restriction of LMI method.

In this paper, we investigate the dynamic Markov jump filters for discrete-time systems subject to random delays, where the delay process is characterized by a finite state Markov chain. In order to solve the estimation problem, the single random delayed measurement is firstly rewritten as a multiplicative-noise stochastic measurement system with constant delays, where the noises are of jump variables and mutually independent. Then, with the application of measurement reorganization, the delayed measurement system is further transformed into two channel delay-free system, and thus the proposed estimation problem can be reformulated as the one for a class of Markov jump linear system without delays [16–19]. Finally, the optimal filter is derived by solving two sets of standard Riccati equations, and the performance is clearly demonstrated through a simulation example.

The remainder of this paper is organized as follows. In Section 2, we present the problem formulation and some assumptions. In Section 3, the recursive filter equations are derived by using the reorganized innovation analysis approach, while a new Markov chain is defined according to the reorganized observations. In Section 4, we give an numerical example to show the efficiency of the proposed algorithms. The paper is concluded in Section 5 with some final comments.

2. Problem formulation

Consider the following discrete-time linear systems with random delayed observations

$$x(k+1) = A(k)x(k) + G(k)w(k), \tag{1}$$

$$y(k) = L(k)x(k-d(k)) + H(k)v(k), \tag{2}$$

where $x(k)$ denotes the $\mathscr{R}^n$-valued state sequence, $w(k)$ and $v(k)$ are random disturbances in $\mathscr{R}^p$ and $\mathscr{R}^q$, respectively, $y(k)$ is the $\mathscr{R}^m$-valued output sequence, and $d(k)$ is the random but bounded time delay of the system. $A(k), G(k), L(k)$ and $H(k)$ are matrices of appropriate dimension. We first make the following assumptions for the above system.

Assumption 1. $w(k)$ and $v(k)$ are null mean second-order, independent wide sense stationary sequences with covariance matrices I_p and I_q, respectively.

Assumption 2. $d(k)$ is a discrete-time Markov chain with finite state space $\{0, d\}$, and transition probability matrix $P = [p_{ij}]$. We set $p_i(k) = p(d(k) = i)$ $(i = 0, d)$ and denote $p_k = [p_0(k)\ p_d(k)]'$, which satisfies the difference equation $p_k = P' p_{k-1}$. Here we assume that $d(k)$ is observable at each sampling time of k.

Assumption 3. The initial condition (x_0, d_0) are such that x_0 and d_0 are independent random variables with $E(x_0) = \mu_0$ and $E(x_0 x_0') = X_0$.

Assumption 4. x_0 and $\{d(k)\}$ are independent of $\{w(k)\}$ and $\{v(k)\}$.

Assumption 5. $H(k)H(k)' > 0$.

Remark 1. In Assumption 2, we model the random delay $d(k)$ as a two state Markov chain. At first, it may seem unrealistic to model the delay as a finite process. However, we note that many continuous state process can be approximated by a finite state process. Further, the Markov model leads to a very nice reformulation of the problem, we believe the assumption is well justified.

The optimal estimation problem can be stated as: *Given the observation sequences* $\{y(0), \ldots, y(k)\}$, *find a dynamic Markov jump filter* $\hat{x}(k)$ *of* $x(k)$.

The reason for choosing this kind of filter is that it depends just on the present value of the Markov parameter. In this paper, we assume that both $y(k)$ and $d(k)$ are directly accessible at each time k, thus the basic Kalman filter result is valid for this case (as shown in [20]). A representation of the Kalman filter is given by

$$\hat{x}^0(k) = \sum_{d:allpaths} \hat{x}(k|k, d(0), \ldots, d(k)) P(d(0), \ldots, d(k)|y(0), \ldots, y(k)). \tag{3}$$

As pointed out in [21], the Kalman filter (3) depends upon past as well as the current values of the jump parameter. It requires exponentially increasing memory and computation in time. To get around this form sample path dependence, the Markov jump filter is considered in this paper, which is designed by reducing the information available to the filter from $\{y(0), \ldots, y(k); d(0), \ldots, d(k)\}$ to $\{y(0), \ldots, y(k); d(k)\}$.

Note that the observation equation is with random jump delays, so the standard Markov jump filtering theory is not applicable to estimate the state directly. State augmentation may be a feasible way to deal with the random delay, which, however, usually leads to the cost of complex computation. Here we employ the reorganized observation method [14] to deal with the random jump delay, which has significant advantages on reducing computation compared with many other approaches (for example, state augmentation method [14]).

3. Markov jump filtering

The purpose of this section is to design the Markov jump filtering for systems (1) and (2). With the reorganized measurement approach, the estimation problem for systems with random jump delays is transformed into the one for a delay-free system with random jump parameters. Then the dynamic Markov jump filters are presented according to the existed theory.