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Towards adaptive topology optimization

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ABSTRACT

This paper presents a new fully-automated adaptation strategy for structural topology optimization (TO) methods. In this work, TO is based on the SIMP method on unstructured tetrahedral meshes. The SIMP density gradient is used to locate solid-void interface and *h*-adaptation is applied for a better definition of this interface and, at the same time, de-refinement is performed to coarsen the mesh in fully solid and void regions. Since the mesh is no longer uniform after such an adaptation, classical filtering techniques have to be revisited to ensure mesh-independency and checkerboard-free designs. Using this adaptive scheme improves the objective function minimization and leads to a higher resolution in the description of the optimal shape boundary (solid-void interface) at a lower computational cost. This paper combines a 3D implementation of the SIMP method for unstructured tetrahedral meshes with an original mesh adaptation strategy. The approach is validated on several examples to illustrate its effectiveness.

1. Introduction

Topology optimization (TO) of continuum structures [1,2] is a powerful tool that gradually becomes a key step in the design process of many products and structures. It consists in calculating the optimal distribution of material, with respect to a given objective, in a *design* domain under sets of constraints. It is becoming a very attractive and important tool in practical engineering [3–5] since it allows producing significantly superior designs and greater savings, than when using parametric optimization or trial and error approaches. Over the past decades, the most widely used TO techniques are the Solid Isotropic Material with Penalization (SIMP) method [6], homogenization based methods [7,8], level sets based methods [9,10] and Evolutionary Structural Optimization (ESO) methods [11,12].

In general terms, the SIMP method consists in determining whether or not a point located inside a given *design* domain should be solid material. In the SIMP method, the volume fraction is prescribed as an input, and is kept constant throughout optimization iterations. Although optimization results are likely to be enhanced through mesh refinement, this refinement is only required close to its boundary, while interior (fully solid) and exterior (fully void) material can be defined using a coarse discretization. These remarks suggest, as pointed out by Aremu et al. [13], that an adaptive mesh improvement strategy could be incorporated in the SIMP optimization process. This paper develops a new adaptive TO scheme which performs both mesh refinement and de-refinement for 3D optimization problems. The mesh adaptation strategy only refines elements at the boundary of the optimal shape and coarsens elements when moving away from it. Using such an adaptive process improves the boundary shape extraction and reduces subsequent post-processing for the interpretation of TO results. Furthermore, by coarsening the mesh inside and outside the optimized domain, the computational cost of SIMP iterations is expected to considerably decrease. Another potential gain that can be foreseen is increasing the achievable geometric complexity of optimal designs. Indeed, a better definition of the boundary and more geometric complexity are attractive when using TO methods along with additive manufacturing. Moreover, as mentioned in [14], improving the quality of TO results at the design stage, even by a very modest amount, may have a huge impact on the overall cost of products when considering the entire lifespan. It is, for example, the case in aerospace applications, for which a tiny loss in weight induces very important fuel savings along the use of these products.

The paper is organized as follows. In the next section (Section 2) we present and discuss previous work related to adaptive TO. Section 3 starts with a short overview of the SIMP method and introduces the concepts on which our mesh adaptation scheme is based. It also shows that classical filtering techniques, that have proven efficiency in preventing numerical instabilities of the SIMP method on uniform meshes, have to be adapted in the context of using highly non-uniform unstructured meshes. Section 3 ends with an analysis of the influence, on optimization results obtained, of the main parameters of our adaptation scheme. In Section 4, the effectiveness of our approach is validated through sets of

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numerical examples. Conclusions are drawn and we sketch some directions for future work in the last section.

2. Related work

Although a limited number of approaches can be found in the literature towards implementing adaptive TO of 3D structures, setting up adaptive schemes to improve optimal solutions is not a new idea. In 1995, Maute and Ramm [15] indeed proposed a method for adapting optimization results in 2D. This methods starts with roughly extracting isodensity curves, which is followed by refining the mesh around these curves and ends with considering this new mesh as a new optimization problem, with the same boundary conditions and loads. By reducing, at each new optimization, the finite element mesh size and increasing the considered value in isodensity curves extraction, several optimization cycles are applied, until reaching a result with satisfying quality. However, this technique is restricted to 2D optimization problems and the overall domain is refined (no de-refinement is carried out). Later, based on the geometrical discretization error, Ramm et al. [16] proposed an *h*-adaptive strategy for shape optimization. As an extension of their method to topology optimization, they were the first, to our knowledge, to introduce the idea that the spatial gradient of material distribution can be used as a refinement criterion in topology optimization. Costa and Alves [17] refined an initially coarse triangular mesh after a certain number of optimization steps. In their work, the refinement strategy is based on estimating the error on stress distribution. After each refinement, a constrained Laplacian smoothing is applied on the mesh to improve elements quality. Still, both solid and boundary elements are refined and no de-refinement is applied. Roman Stainko [18] first considered only refining the mesh at the solid-void interface and applying it to tetrahedral meshes. In his approach, new elements are inserted only at the boundary of the optimal design and fully solid and void regions are not refined. A filter is introduced in the refinement process to control the number of elements around the solid-void interface and refinement is performed through partial re-meshing. However, no de-refinement is performed. Few years later, Sturler et al. [19] and Wang et al. [20] introduced the AMR (Adaptive Mesh Refinement) concept to achieve the design that would be obtained on a uniformly fine mesh. Their dynamic mesh refinement and de-refinement is performed continuously throughout the optimization process, but refinement and de-refinement are not carried out simultaneously. Moreover, both solid-void interface and solid volume are refined, which is likely to refine the mesh where it doesn't necessarily need to be. Bruggi et al. [21] propose a modified version of this approach that is driven by two error estimators and for which de-refinement is only performed inside the void region and at the last adaptive step. In this approach, the first error estimator used is related to the amplitude of intermediate regions (solid-void interface) and the second one is related to the FEA error itself.

More recently, Duan et al. [22] introduce an adaptive mesh refinement strategy for TO applied to computational fluid dynamics (CFD) for which refinement is driven by distance to the boundary. Wang et al. [23] propose using two separate discretizations along TO: one discretization for approximating the relative density field and the other one for FEA. The relative density mesh is refined with respect to optimization results while the analysis mesh is refined with respect to FEA error estimation. The advantage is that it guarantees that refinement based on the relative density field does not degrade analysis results. However, the method does not feature any de-refinement process and requires using non-conventional finite elements (so-called *level-one mesh incompatibility* [20]). Being able to use conventional finite elements is indeed an advantage since it basically allows using any FEA solver. Lin et al. [24] performed a two-stage TO algorithm for homogenization methods where the optimal design obtained at the end of the first optimization stage is projected on a refined uniform mesh, which is then used as the initial topology for the second stage of the optimization. Another alternative to improve TO results by combining adaptive meshing strategy and (Bi) directional ESO (so-called BESO) scheme is proposed by Aremu et al. [25] to solve a standard cantilever beam. During the re-meshing process, elements are refined using two refinement templates by bisection and mesh coarsening is performed through edge collapsing. However, this strategy degrades mesh quality and it may not be straightforward to extend it to 3D optimization problems. Similarly, Liu et al. [26] couples an adaptive moving mesh method (also called *r*-adaptivity scheme) with a level set TO scheme. Unlike previous re-meshing and updating procedures, they updated mesh grid points following topology changes to better approximate the optimal shape without changing the mesh topology. Furthermore, large deformation problems with meshfree analysis in TO design have been addressed by He et al. [27]. They have obtained good optimal solutions without mesh difficulties and numerical instabilities by using an element-free Galerkin method, which represents a promising and interesting alternative to FEA based optimization.

Topology optimization capabilities have been successfully implemented in several commercial systems [28, 29]. TOSCA is a commercial TO solution that features an adaptive mesh refinement during the TO process. However, this commercial implementation is limited [14]. In the current version, the adaptive process is limited to two refinement iterations and mesh refinement is implemented using templates based on 2D quad elements that are not straightforward to extend to the general context of unstructured tetrahedral meshes.

As a conclusion, the aforementioned methods are either limited to 2D optimization problems and/or do not feature a de-refinement process. It also appears that, since mesh density strongly influences both the computational cost of TO and the smoothness and accuracy in the description of the optimal shape, mesh adaptation is a potentially very useful tool in the improvement of TO results. This is why, in this paper, we present a fully-automated adaptive TO scheme that includes simultaneous mesh refinement and de-refinement and that applies on 3D unstructured tetrahedral meshes.

3. A new adaptive topology optimization process

As introduced in the previous section, our objective is simultaneously refining and de-refining specified regions of finite element meshes used in TO to improve accuracy in the definition of optimal shape boundary at a low computational cost.

3.1. The SIMP method

3.1.1. Basic principles

The SIMP (Solid Isotropic Material with Penalization) method is based on considering an artificial solid isotropic material with power-law penalization of intermediate densities to steer the optimal solution to a discrete 0-1 design (0 density stands for void and 1 density for full material). Thus, from SIMP results (a distribution of density across the mesh that represents design material) the optimal design shape is obtained from sets of finite elements with a density that is close to 1. In this paper, the implementation of the SIMP method used consists in optimizing the distribution of a fixed amount of material inside the *design* volume towards minimizing its global compliance (or flexibility), which generally means maximizing its stiffness. The distribution of material inside the *design* volume is represented by a relative density distribution $\rho(x, y, z)$, which is updated along finite element iterations. Convergence of