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# Steady-state analysis of constrained normalized adaptive filters for MAI reduction by energy conservation arguments

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**Abstract**

We derive the steady-state performance of a common class of adaptive filters for multiple access interference (MAI) reduction in code division multiple access (CDMA) systems. The adaptive filters under study utilize estimates of the desired user's amplitude and are divided into three groups of least-mean-square (LMS) algorithms differing by the choice of the normalization factor. The steady-state performance is deduced from energy-conservation relations that include a possibly erroneous estimate of the desired user's amplitude. The analyses show that blind algorithms using information about the desired user's amplitude achieve similar performance to that of nonblind algorithms. In addition, geometric considerations reveal the conditions under which the choice of the normalization factor is expected to have great impact on the convergence properties of the algorithms. Numerical simulations show good agreement with theory.


*Keywords:* Multiuser interference; Energy conservation relations; Steady-state analysis

## 1. Introduction

In direct sequence/code division multiple access systems (DS/CDMA), multiple users share the same channel at the same time. Hence, at the receiver the detection of the transmitted symbols of a desired user is strongly affected by the interference originated from the other users in the system. This interference is known as multiple access interference (MAI). Without proper mitigation, this MAI causes severe performance degradation in the detection of the transmitted information.

In multiuser systems, a receiver that achieves the minimum bit error rate (BER) generally demands high computational complexity [1]. Therefore, linear adaptive filters with low computational complexity have been introduced to mitigate MAI [2]. Linear adaptive filters try to approximate a filter that minimizes (or maximizes) a mathematically tractable cost function closely related to the BER. Usually, these adaptive filters track the filter maximizing the signal-to-interference-noise ratio (SINR) [2–13], and the main difference between them lies in the choice of the adaptive algorithm.

In particular, a great deal of effort has been devoted to the study of blind linear receivers based on the recursive least-squares (RLS) and least-mean-square (LMS) algorithms. RLS-based receivers show good convergence speed [10], but they

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demand higher computational complexity than LMS-based receivers and often suffer from numerical instabilities [8] caused by the inherent matrix inversion and possible model mismatch. Moreover, for this particular application, conventional fast versions with linear complexity of the RLS algorithm are hard to be applied because there is no time-index-shifting property of the input data [10]. Furthermore, in relatively fast time-varying scenarios, LMS-based receivers can outperform RLS-based receivers [13] because the tracking behavior of the LMS algorithm is usually superior to that of the RLS algorithm in nonstationary environments [14]. In this study, the focus is on LMS-based receivers.

When a training sequence is necessary for the filter adaptation in a LMS-based receiver, the LMS algorithm is said to be nonblind; otherwise, it is said to be blind. Blind LMS algorithms are preferable because the overhead imposed by training sequences is not present, so the throughput of the system is potentially higher. They are usually obtained by restricting the adaptive filter to satisfy a constraint determined by the desired user's signature. A celebrated blind LMS algorithm for MAI mitigation has been proposed in [2]. Unfortunately, the steady-state performance achieved with this approach is much worse than the performance achieved with nonblind algorithms, especially when the level of background noise is small. This performance difference between blind and nonblind algorithms can be reduced if estimates of the desired user's amplitude and transmitted symbol are utilized effectively in the blind algorithms [4,9,12,15].

We can further divide LMS algorithms into non-normalized algorithms [2,9] and normalized algorithms [4,5,9,11–13]. Normalized LMS algorithms are frequently used because they are potentially faster than non-normalized LMS algorithms for both uncorrelated and correlated input data [16]. The work in [9] introduces a normalization factor that reduces the sensitivity of LMS algorithms to changes in the number of users and/or the signal-to-noise ratio (SNR). In [4] the normalization factor has been derived from projections onto closed nonconvex sets. By considering geometric properties of projections onto closed convex sets, [5,12,15] have proposed a normalization factor aiming at fast convergence speed.

In this study, we deduce closed-form expressions for the steady-state performance of constrained normalized LMS algorithms. The closed-form expressions are derived by including the possible information about the desired user's amplitude into energy-conservation relations [17–19]. More precisely, we focus on the steady-state performance of the algorithms introduced in [4,5,9,12]. These algorithms are examples of a more general class of receivers presented in [13], which in turn is based on the adaptive projected subgradient method [20]. For simplicity, we divide the algorithms into three different groups according to their normalization factors (cf. Table 1):

- Group I [4]: the OPM-based gradient projection (OPM-GP) algorithm, the generalized projection (GP) algorithm, and the space alternating generalized projection (SAGP) algorithm;

Table 1
Characterization of the adaptive filters under study

| Group | Algorithm | $\alpha$ | $\tilde{b}_1[i]$ | $g(\boldsymbol{r}[i])$ |
|---|---|---|---|---|
| I | GP [4] | $A_1$ | $\mathrm{sgn}(\boldsymbol{h}_{i-1}^{\mathrm{T}}\boldsymbol{r}[i])$ | $\lVert\boldsymbol{r}[i]\rVert^2$ |
| I | SAGP [4] | $\tilde{A}_1$ | $\mathrm{sgn}(\boldsymbol{h}_{i-1}^{\mathrm{T}}\boldsymbol{r}[i])$ | $\lVert\boldsymbol{r}[i]\rVert^2$ |
| I | OPM-GP [4] | 0 | - | $\lVert\boldsymbol{r}[i]\rVert^2$ |
| II | Modified GP | $A_1$ | $\mathrm{sgn}(\boldsymbol{h}_{i-1}^{\mathrm{T}}\boldsymbol{r}[i])$ | $\boldsymbol{r}[i]^{\mathrm{T}}\boldsymbol{P}\boldsymbol{r}[i]$ |
| II | Modified SAGP [12] | $\tilde{A}_1$ | $\mathrm{sgn}(\boldsymbol{h}_{i-1}^{\mathrm{T}}\boldsymbol{r}[i])$ | $\boldsymbol{r}[i]^{\mathrm{T}}\boldsymbol{P}\boldsymbol{r}[i]$ |
| II | Modified OPM-GP [5] | 0 | – | $\boldsymbol{r}[i]^{\mathrm{T}}\boldsymbol{P}\boldsymbol{r}[i]$ |
| III | BLMS (normalized)[9] | 0 | – | $\beta[i]=(1-\kappa)\beta[i-1]+\kappa\lVert\boldsymbol{r}[i]\rVert^2$, $\beta[0]=\lVert\boldsymbol{r}[0]\rVert^2$, $0<\kappa<1$ |
| III | CLMS-AE (normalized)[9] | $\tilde{A}_1$ | $b_1[i]$ | $\beta[i]=(1-\kappa)\beta[i-1]+\kappa\lVert\boldsymbol{r}[i]\rVert^2$, $\beta[0]=\lVert\boldsymbol{r}[0]\rVert^2$, $0<\kappa<1$ |
| III | DD-CLMS-AE (normalized)[9] | $\tilde{A}_1$ | $\mathrm{sgn}(\boldsymbol{h}_{i-1}^{\mathrm{T}}\boldsymbol{r}[i])$ | $\beta[i]=(1-\kappa)\beta[i-1]+\kappa\lVert\boldsymbol{r}[i]\rVert^2$, $\beta[0]=\lVert\boldsymbol{r}[0]\rVert^2$, $0<\kappa<1$ |