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Efficient methodologies for reliability-based design optimization of composite panels

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ABSTRACT

The main factors governing the design of composite laminates are the geometrical dimensions, the stacking sequence –including ply thickness and orientation angles–, the mechanical properties of the materials, the applied loads and the performance requirements. Most of these factors are commonly affected by uncertainty and this should be taken into account when designing these structures. Thus, uncertainty quantification should be used to evaluate the performance requirements and a reliability-based procedure is advisable when the design is optimized. However, these methods present several drawbacks, like the lack of trustworthy information about the uncertainty present in the variables of the model or the high computational cost required to apply the algorithms to medium to large models. This paper evaluates several methodologies for design optimization of composite panels under uncertainty. The uncertainty quantification is performed using stochastic expansion and limit state approximation methods. Monte Carlo sampling is also used to verify reliability results. The optimization process is carried out using gradient-based and genetic algorithms, with either continuous or discrete design variables. Surrogate methods, including polynomial, kriging, multivariate adaptive regression splines, and artificial neural networks, as well as parallel computing, have been leveraged to keep analysis times under acceptable levels. An application example of a stiffened composite panel of an aircraft fuselage is presented to demonstrate the computational performance and the accuracy of the methods. Results show major improvement in analysis time without compromising on precision.

1. Introduction

Design optimization and reliability analysis are useful techniques that improve structural performance when applied to the design of composite panels. A recent review of selected methodologies and a summary of current developments in this field can be found in [1].

The first works about design optimization of composites in presence of uncertainty were published in the 1990's [2–5]. Since then, several authors have proposed new optimization algorithms or modifications to existing ones which can be successfully applied to optimize composite laminates when uncertainty is considered [6–9]. As discrete design variables are usually required for optimization of the stacking sequence, most of the contributions involve metaheuristic methods, like genetic algorithms [10] or particle swarm optimization [11,12], which are particularly suited for discrete problems.

However, these methodologies are very time-consuming and, except in simple cases, they often reach an unacceptable computational cost, because multiple evaluations of implicit functions are required to obtain the structural response. This is the one of the reasons why reliability-based design optimization (RBDO) is not a mainstream design technique in the field of composite structures and most of the existent design procedures are based on semi-deterministic approaches [13,14]. Surrogate methods [15–17] allow the transformation of a complex implicit model into an analytic approximation that decreases in several orders of magnitude the computational cost without a significant loss of accuracy. Parallel computing [18,19] can be used to further decrease the computational cost when evaluating the surrogate approximation.

Other researchers have already applied surrogate models to the design optimization of composite structures. Rais-Rohani and Singh [20] considered global and local polynomial surrogates, combined with the first order reliability method. Artificial neural networks have also been applied in this field, see for instance [21].

The objective of this work is the application of different reliability analysis and optimization algorithms, combined with surrogate methods and parallel computing, in order to establish some efficient methodologies that make possible the application of RBDO procedures to complex composite structures with a large number of degrees of freedom and several design and random variables. Thus,

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several up-to-date reliability methods and surrogate models have been applied to a real design case of an aircraft structure, consisting of a curved stiffened panel of a fuselage composed by a carbon/epoxy composite skin stiffened by aluminum ribs and stringers. Two structural limit state functions –buckling and Tsai–Wu failure stress criterion– have been considered with a serial failure mechanism. The efficiency and accuracy of the results obtained with different methods is studied and compared.

The RBDO problem is carried out by the performance measure approach and reliability analysis is performed using the first order reliability method, polynomial chaos expansion and stochastic collocation. Monte Carlo sampling is also used to verify reliability results. The optimization process is carried out using sequential quadratic programming when considering continuous design variables and genetic algorithms when dealing with discrete design variables. The surrogate models are built using polynomial, kriging, multivariate adaptive regression splines, and artificial neural networks. In the next sections the formulation is presented and the methodologies are applied to a demonstration example.

2. Reliability analysis

In engineering design, structural safety is usually verified by means of semi-deterministic procedures with safety coefficients that estimate the effects of uncertainty in the structural system, increasing the value of loads and decreasing the level of the material strength. In relation to composites, there are several national and international standards for the design of certain classes of composite structures that make use of safety coefficients [13,14].

Reliability analysis methods assess the structural safety taking into account the random nature of all phenomena affecting to a structural system [22]. In these methods, the design region is divided by a limit state function $g(\mathbf{a})$ as

$$\text{Failure domain: } F = \{\mathbf{a} \mid g(\mathbf{a}) < 0\}, \tag{1}$$

$$\text{Safety domain: } S = \{\mathbf{a} \mid g(\mathbf{a}) \geqslant 0\}. \tag{2}$$

The boundary between failure and safety domains is the failure surface or limit state surface, which generally is a hypersurface of $n-1$ dimensions in the n-dimensional space of random variables $\boldsymbol{a}$. According to this, the probability of failure p_f is formulated as

$$p_f = P[g(\boldsymbol{a}) < 0] = \int_{g(\boldsymbol{a})<0} \cdots \int f_{\boldsymbol{A}}(\boldsymbol{a}) d\boldsymbol{a}, \tag{3}$$

where $f_{\boldsymbol{A}}(\boldsymbol{a})$ is the joint probability density function of the random variables. Except in some particular cases, the integral expression (3) cannot be resolved analytically, because of the nonlinearity of $f_{\boldsymbol{A}}(\boldsymbol{a})$ and also due to the fact that the number of random variables usually employed is large, and therefore the dimension of the problem. Besides, the joint probability density function is usually not available. In consequence, alternative methods have to be used to solve (3). In this work, different methods of reliability analysis are applied to evaluate the uncertainty constraints within an optimization procedure. Some of these methods and their implementation have been studied by the authors in the past [23,24], demonstrating a good performance and giving accurate results. The methods are briefly described next.

2.1. First order reliability method

The FORM or first order reliability method [25] approximates the limit state function by the Taylor series expansion of first order at the most probable point of failure (MPP) $\boldsymbol{a}'_f$, which is the minimum distance point on the limit state surface from the origin of the coordinate system in the standardized domain of the random variables $\boldsymbol{a}'$:

$$g'(\boldsymbol{a}') \simeq g(\boldsymbol{a}'_f) + \nabla g(\boldsymbol{a}'_f)^T (\boldsymbol{a}' - \boldsymbol{a}'_f). \tag{4}$$

The reliability index is invariant with respect to the formulation of the limit state function and is obtained as

$$\beta = -\frac{\boldsymbol{a}'^T_f \nabla g(\boldsymbol{a}'_f)}{\sqrt{\nabla g(\boldsymbol{a}'_f)^T \nabla g(\boldsymbol{a}'_f)}}. \tag{5}$$

In order to use non-Gaussian random variables, a transformation needs to be applied. In this work, we have employed the Nataf transformation [26]. This transformation only needs the marginal cumulative distribution function $F_{A_i}(A_i)$ and the correlation matrix of the variables C_A. Thus, transformed variables can be expressed in the normal space as

$$a'_i = \Phi^{-1}\left[F_{A_i}(a_i)\right] \tag{6}$$

where Φ is the cumulative distribution function of a standard normal random variable. The joint probability density function ϕ_n can be characterized in the space of transformed variables as

$$\phi_n\left(a', C_A\right) = \frac{1}{\sqrt{(2\pi)^n |C_{A'}|}} exp\left[-\frac{1}{2} a'^T C_{A'}^{-1} a'\right]. \tag{7}$$

2.2. Stochastic expansion methods

The stochastic expansion methods are based on the work by [27]. These methods employ the concepts of projection, orthogonality and weak convergence by means of multidimensional polynomial approximations. The final solution is expressed as a functional mapping, instead of a set of statistics. In this paper, polynomial chaos expansion and stochastic collocation have been considered [28,29].

Polynomial chaos expansion (PCE) is based on multidimensional orthogonal polynomials to approximate the functional form between the stochastic response output and each one of the random inputs. The Wiener–Askey scheme [30,31] is used in this work. A simple definition of the PCE for a Gaussian random response R as a convergent series is as follows

$$\begin{aligned} R(\boldsymbol{a}) &= \theta_0 \Gamma_0 + \sum_{i_1=1}^{\infty} \theta_{i_1} \Gamma_1\left(\xi_{i_1}(\boldsymbol{a})\right) \\ &+ \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{i_1} \theta_{i_1 i_2} \Gamma_2\left(\xi_{i_1}(\boldsymbol{a}), \xi_{i_2}(\boldsymbol{a})\right) \\ &+ \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{i_1} \sum_{i_3=1}^{i_2} \theta_{i_1 i_2 i_3} \Gamma_3\left(\xi_{i_1}(\boldsymbol{a}), \xi_{i_2}(\boldsymbol{a}), \xi_{i_3}(\boldsymbol{a})\right) + \cdots \end{aligned} \tag{8}$$

where $\{\xi_i(\boldsymbol{a})\}_{i=1}^{\infty}$ is a set of Gaussian random variables, $\Gamma_p\left(\xi_{i1}, \ldots, \xi_{ip}\right)$ is the generic element of a set of multidimensional Hermite polynomials, and $\theta_{i1}, \ldots, \theta_{ip}$ are deterministic constants. Eq. (8) can be written more simply as

$$R(\boldsymbol{a}) = \sum_{i=0}^{P} b_i \Psi_i\left[\boldsymbol{\xi}(\boldsymbol{a})\right], \tag{9}$$

where b_i and $\Psi_i[\boldsymbol{\xi}(\boldsymbol{a})]$ are one-to-one correspondences between the coefficients $\theta_{i1}, \ldots, \theta_{ip}$ and the functions $\Gamma_p(\xi_{i1}, ..., \xi_{ip})$, respectively. The general expression to obtain the Hermite polynomials [32] is formulated as

$$\Gamma_p\left(\xi_{i1}, \ldots, \xi_{ip}\right) = (-1)^n \frac{\partial^n e^{-\frac{1}{2}\xi^T \xi}}{\partial \xi_{i1}, \ldots, \partial \xi_{ip}} e^{\frac{1}{2}\xi^T \xi}. \tag{10}$$

Stochastic collocation (SC) [33] is a stochastic expansion approach that is closely related to PCE. It is defined as a sum of multidimensional Langrange interpolation polynomials, considering one polynomial per collocation point. The coefficients of the expansion are the response values at each collocation point, and can be formulated as