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Characterizing thermo-mechanical behavior of superalloy using the eigenfunction virtual fields method

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ABSTRACT

An innovative procedure for characterizing thermo-mechanical behavior of superalloy using the Eigenfunction Virtual Fields Method (EVFM) was proposed. First, the principle of EVFM for thermo-mechanical constants identification was developed based on the principal components analysis. Then, the strain fields were extracted from finite element (FE) simulation of a superalloy plate with a circular hole under uniaxial tension and uniform temperature increment. In addition, the virtual fields were constructed using the eigenvectors of augmented strain matrix. Finally, the thermo-mechanical constants were inversed from the strain fields, and the effect of mesh size and noise on the inversion results was analyzed. The results show that the thermo-mechanical constants of superalloy inversed from EVFM using these eigenvectors as virtual fields are in excellent agreement with the true values.

1. Introduction

Superalloy materials are widely used in aviation and aerospace fields as engine materials due to its excellent high temperature thermo-mechanical behavior [1,2]. However, how to characterize the thermo-mechanical constants of superalloy materials correctly, accurately and simply is an important and difficult problem. Virtual Fields Method (VFM), which was proposed by Grédiac and Pierron [3,4], provides an effective tool for high temperature parameter identification of materials. VFM brings the advantage of full-field and non-contact of optical measurement into play.

VFM has been commonly applied in many areas since it was proposed. Among these applications, the elastic stiffness identification is one of the most interested fields. Grédiac et al. applied VFM to the identification of hyperelastic parameters [5], elasto-plastic constitutive parameters [6], bending rigidities of anisotropic plates [7]. Pierron et al. extended the virtual fields method to elasto-plastic material identification with cyclic loads and kinematic hardening [8] and identification of elasto-visco-plastic parameters [9]. Rahmani et al. used VFM to mechanical properties identification of 3D particulate composites [10] and proposed a Regularized Virtual Fields Method (RVFM) to characterize mechanical properties of composite materials [11]. Xie et al. applied VFM to elastic stiffness identification of composite materials [12] and 3D printing material [13]. Ma et al. [14] used VFM to inverse and decouple thermo-mechanical deformation of anisotropic materials under high-temperature environments. Furthermore, Gao and Shang [15] proposed a Deformation-Pattern-based Digital Image Correlation (DPDIC) method to measure directly residual stresses by digital image correlation using hole drilling. The deformation pattern that was governed by the residual stresses was used to affine transform the image captured after the object was deformed. Liu and Gao [16] extended DPDIC to measure coefficient of thermal expansion (CTE) of film, and the results of CTEs from DPDIC and conventional DIC methods were compared with the actual CTE, showing an improved accuracy.

Recently, Subramanian et al. [17] proposed an Eigenfunction Virtual Fields Method (EVFM), which systematically identifies virtual fields by performing a principal component analysis (PCA) of the strain field measured from experiments. The virtual strain components to be used in VFM are then chosen to be the eigenfunctions. In addition to being a physically meaningful set of virtual fields, such a choice exploits the orthogonality of the computed eigenfunctions while simultaneously eliminating computation of a large number of coefficients that define the virtual fields in prior approaches. They applied EVFM to homogeneous linear elastic property evaluation [17], computation of elastic constants of functionally graded materials [18] and identification of orthotropic elastic constants [19,20].

In the present work, the EVFM was extended to thermo-mechanical parameters identification of superalloy materials under high temperature environment. The principle of EVFM for

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thermo-mechanical parameters identification was presented firstly. Then, a finite element simulation was carried out on a superalloy plate with a circular hole under uniaxial tension and uniform temperature increment. The eigenfunctions calculated from the strain fields were selected as virtual fields to inverse thermo-mechanical parameters. Finally, the effect of mesh size and noise on the inversion results was analyzed.

2. Principle of EVFM

2.1. Computation of eigenfunctions of strain fields

EVFM is based on the eigenfunctions of strain field, and the strain field usually calculated from full-field optical method, such as digital image correlation, on a grid of $m\times n$ points. Two augmented matrices E^r and E^c of sizes $3m\times n$ and $m\times 3n$ respectively are constructed from the strain component grids as follows [17]:

$$E^r=\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}; E^c=\begin{bmatrix} E_1 & E_2 & E_3 \end{bmatrix} \tag{1}$$

where E_1, E_2 and E_3 are the strain components on the grid points. Singular Value Decomposition (SVD) is performed on the E^r and E^c, and the eigenfunctions are obtained by Principal Component Analysis (PCA). The decompositions are

$$E^r_{(3m\times n)}=L^r_{(3m\times 3m)}S^r_{(3m\times n)}(R^r)^T_{(n\times n)} \tag{2}$$

$$E^c_{(m\times 3n)}=L^c_{(m\times m)}S^c_{(m\times 3n)}(R^c)^T_{(3n\times 3n)} \tag{3}$$

where the columns of L^r and L^c contain the left eigenvectors of E^r and E^c; S^r and S^c are diagonal matrices that contain the singular values of E^r and E^c; the columns of R^r and R^c contain the right eigenvectors of E^r and E^c, respectively. A significant advantage of this decomposition is that each group of eigenvectors is a complete orthonormal basis; therefore, the right eigenvectors are orthonormal basis for the row space of each augmented matrix, as well as the left eigenvectors are orthonormal basis for the column space [17].

It is proved that full-field strain data are highly redundant and only a small number (p) eigenvectors are dominant. Thus, it is adequate to reconstruct the strain matrices in terms of the p-dimensional subspace of the row and column spaces spanned by these p dominant left and right eigenvectors. If $\overline{E}_1^r$ is the reconstructed matrix of strain values E_1 using the p right eigenvectors of E^r, then

$$\overline{E}_1^r=\overline{A}_1^r\left(\overline{R}^r\right)^T=E_1\overline{R}^r\left(\overline{R}^r\right)^T \tag{4}$$

where $\overline{R}^r$ is the $n\times p$ matrix whose columns contain the p right dominant eigenvectors of E^r; and $\overline{A}_1^r=E_1\overline{R}^r$ is an $m\times p$ matrix, which contains the rows of E_1 along the columns of $\overline{R}^r$. It is evident that the kth row $(\overline{E}_1^r)^{(k,-)}$ of $\overline{E}_1^r$ can be obtained as

$$\left(\overline{E}_1^r\right)^{(k,-)}=\left(\overline{A}_1^r\right)^{(k,-)}\left(\overline{R}^r\right)^T \tag{5}$$

In the same way, $\overline{E}_1^c$ is the reconstructed matrix of E_1 from the p dominant left eigenvectors of E^c, which is given by

$$\overline{E}_1^c=\overline{L}^c\left(\overline{A}_1^c\right)^T=\overline{L}^c\left(\overline{L}^c\right)^TE_1 \tag{6}$$

where $\overline{L}^c$ is the $m\times p$ matrix whose columns contain the p left eigenvectors of E^c; and $\overline{A}_1^c=(E_1)^T\overline{L}^c$ is an $p\times n$ matrix, which contains the columns of E_1 along the columns of $\overline{L}^c$. The other two strain component matrices are reconstructed similarly.

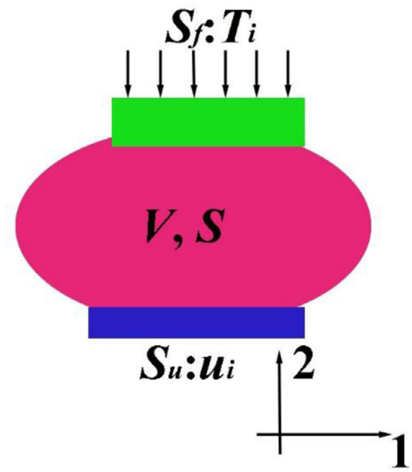

Fig. 1. Superalloy materials of any shape under in-plane thermo-mechanical loading.

2.2. EVFM for characterizing thermo-mechanical behavior of superalloy

As shown in Fig. 1, a solid with constant thickness of any shape subjected to in-plane thermo-mechanical loading. Here, V is the volume occupied by the solid of interest, and S is the exterior surface of the solid. S_u and S_f are the displacement boundary and the loading boundary of the external surface area, respectively. u_i (i = 1,2) is the displacement over the displacement boundary. T_i (i = 1,2) is the force per unit area over the loading boundary. Thus, the governing equation of virtual work can be expressed as [3,14]

$$\int_V \sigma_i\varepsilon_i^* dV=\int_{S_f} T_iu_i^* dS \tag{7}$$

where u_i^*(i=1,2) is any kinematically admissible virtual displacement field; ε_i^*(i=1,2,12) is the virtual strain; σ_i(i=1,2,12) is the stress.

Under the thermo-mechanical loading conditions, the stress-strain relations of isotropic superalloy materials can be expressed as follows:

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix}=\begin{bmatrix} Q_1 & Q_2 & 0 \\ Q_2 & Q_1 & 0 \\ 0 & 0 & Q_{12} \end{bmatrix}\begin{pmatrix} \varepsilon_1-\alpha\Delta T \\ \varepsilon_2-\alpha\Delta T \\ \varepsilon_{12} \end{pmatrix} \tag{8}$$

where $Q_1=\frac{E}{1-\mu^2}$, $Q_2=\mu Q_1=\frac{E\mu}{1-\mu^2}$, $Q_{12}=\frac{Q_1-Q_2}{2}=G=\frac{E}{2(1+\mu)}$ are the coefficients of the stiffness matrix. α is the thermal expansion coefficients of the isotropic superalloy materials. ε_i(i=1,2,12) is the true strain field. $\Delta T(1,2)$ is the temperature increment in the coordinate (1,2). Then, Eq. (7) can be expressed as

$$\begin{aligned} &h\int_S \varepsilon_1^*[Q_1(\varepsilon_1-\alpha\Delta T)+Q_2(\varepsilon_2-\alpha\Delta T)]dS+ \\ &h\int_S \varepsilon_2^*[Q_2(\varepsilon_1-\alpha\Delta T)+Q_1(\varepsilon_2-\alpha\Delta T)]dS+ \\ &h\int_S \varepsilon_{12}^*[Q_{12}\varepsilon_{12}]dS=\int_{S_f}\left(T_1u_1^*+T_2u_2^*\right)dS \end{aligned} \tag{9}$$

where h is the thickness of the solid.

Eq. (9) can be simplified as

$$\begin{aligned} &\tfrac{Eh}{1-\mu^2}\int_S \varepsilon_1^*[(\varepsilon_1+\mu\varepsilon_2)-(1+\mu)\alpha\Delta T]dS+ \\ &\tfrac{Eh}{1-\mu^2}\int_S \varepsilon_2^*[(\mu\varepsilon_1+\varepsilon_2)-(1+\mu)\alpha\Delta T]dS+ \\ &\tfrac{Eh}{2(1+\mu)}\int_S \varepsilon_{12}^*\varepsilon_{12}dS=\int_{S_f}\left(T_1u_1^*+T_2u_2^*\right)dS \end{aligned} \tag{10}$$

2.3. Finite element simulation

Since the primary objective of this paper is to demonstrate the accuracy of EVFM for thermo-mechanical properties, strain fields from