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A low-complexity suboptimal filter for continuous–discrete linear systems with parametric uncertainties

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Abstract

We present a novel suboptimal filtering algorithm addressing estimation problems that arise in mixed continuous–discrete linear time-varying systems with stochastic parametric uncertainties. The suboptimal state estimate is formed by summing of local Kalman estimates with weights depending only on time instants t_k. In contrast to optimal weights, the suboptimal weights do not depend on current measurements, and thus the proposed filter is of a low-complexity and it can easily be implemented in real-time. High accuracy and efficiency of the suboptimal filter are demonstrated on the damper harmonic oscillator motion and the vehicle motion constrained to a plane.

Keywords: Adaptive filtering; Kalman filtering; Continuous–discrete system; Mean-square error; Fusion

1. Introduction

We consider a linear system described by the stochastic differential equation

$$\dot{x}_t = F_t(\theta)x_t + G_t(\theta)v_t, \quad t \geqslant 0, \tag{1}$$

where $x_t \in \mathbf{R}^n$ is the state, $v_t \in \mathbf{R}^q$ is a zero-mean Gaussian white noise with covariance $E(v_t v_s^{\mathrm{T}}) = Q_t\delta(t-s)$, and $F_t \in \mathbf{R}^{n\times n}$, $G_t \in \mathbf{R}^{n\times q}$, and $Q_t \in \mathbf{R}^{q\times q}$.

Discrete linear measurements are taken at time instants t_k:

$$y_{t_k} = H_{t_k}(\theta)x_{t_k} + w_{t_k},$$
$$k = 1,2,\ldots, \quad t_{k+1} > t_k \geqslant t_0 = 0, \tag{2}$$

where $y_{t_k} \in \mathbf{R}^m$ is the measurement $H_{t_k} \in \mathbf{R}^{n\times m}$, and $\{w_{t_k} \in \mathbf{R}^m, k = 1,2,\ldots\}$ is a zero-mean white Gaussian sequence, $w_{t_k} \sim \mathrm{N}(0, R_{t_k}(\theta))$. The distribution of the initial state x_0 is Gaussian, $x_0 \sim \mathrm{N}(\bar{x_0}(\theta), P_0(\theta))$, and x_0, v_t, and $\{w_{t_k}\}$ are assumed independent. The sample times $\{t_k\}$ are scheduled and completely known in advance. In general, the partition of the time instants $\triangle t_k = t_k - t_{k-1}$ is an arbitrary.

In addition, it is assumed that the matrices $F_t(\theta)$, $G_t(\theta)$, $Q_t(\theta)$, $H_{t_k}(\theta)$, $R_{t_k}(\theta)$, $P_0(\theta)$ and the initial mean $\bar{x}_0(\theta)$ include the unknown parameter $\theta \in \mathbf{R}^r$, which takes only a finite set of values

$$\theta \in \{\theta_1, \ldots, \theta_N\}. \tag{3}$$

This finite set might be a result of discretizing a continuous parameter space [1,2]. The parameter θ is time-invariant, that is at the starting point $t = 0$

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Nomenclature

$\mathbf{R}^n$	Euclidean space of dimension **n**
$\mathbf{R}^{n\times m}$	space of $n \times m$ real matrices
θ	unknown parameter (vector), $\theta \in \mathbf{R}^r$
θ_i	value of parameter θ, $\theta = \theta_i$, $i = 1, \ldots, N$
N	total number of values of parameter $\theta = \{\theta, \ldots, \theta_N\}$
$x_t = x_t(\theta)$	state vector at time instant **t** depending on θ
$x_t^{(i)} = x_t(\theta_i)$	state vector $x_t = x_t(\theta)$ at the value of parameter $\theta = \theta_i$
y_{t_k}	measurement vector at time instant t_k
y_0^t	collection of all measurements up to time **t**
$p(\theta)$	prior probability of θ
$p(\theta \vert y_0^t)$	a posteriori probability of θ given y_0^t
v_t	system noise vector at time instant **t**, $v_t \in \mathbf{R}^q$
w_{t_k}	measurement noise vector at time instant t_k, $w_{t_k} \in \mathbf{R}^m$
$H_{t_k}(\theta)$	measurement matrix at time instant t_k depending on parameter θ, $H_{t_k}(\theta) \in \mathbf{R}^{n\times m}$
$F_t(\theta)$	system matrix at time instant **t** depending on parameter θ, $F_t(\theta) \in \mathbf{R}^{n\times n}$
$Q_t(\theta)$	covariance of system noise depending on parameter θ, $Q_t \in \mathbf{R}^{q\times q}$
$R_{t_k}(\theta)$	covariance of measurement noise depending on parameter θ, $R_{t_k} \in \mathbf{R}^{m\times m}$
$\hat{x}_t^{\text{opt}}$, P_t^{opt}	optimal mean-square estimate of state x_t given y_0^t and corresponding covariance
$\hat{x}_t^{\text{sub}}$, P_t^{sub}	suboptimal (fusion) estimate of state x_t given y_0^t and corresponding covariance
$\hat{x}_t^{(i)}$, $P_t^{(i)}$	optimal local estimate of state $x_t^{(i)}$ and corresponding covariance
$\hat{x}_{t_k}^{(i)-}$, $P_{t_k}^{(i)-}$	predicted local estimate of state $x_t^{(i)}$ at time t_k and corresponding covariance
$\mathbf{N}(m, P)$	multidimensional normal pdf with mean **m** and P covariance
δ_{ij}	Kronecker delta function
$\delta(t-s)$	Dirac delta function

the parameter is settled, $\theta = \theta_i$, and it cannot change during time progress $t > 0$.

A fundamental problem associated with such systems is estimation of the state x_t from the noisy measurements $y_0^t = \{y_{t_s} : 0 \leqslant t_s \leqslant t\}$.

Many approaches are available for the adaptation of systems. Most identification approaches (see, for example, [1–7]) may be applied to construct an adaptive mechanism. Among existing methods, we are particularly interested in the partitioned adaptive approach that is mathematically based on the Bayesian estimation theory, since it is useful not only for identifying noise statistics but also for estimating unknown system parameters and states, which is sometimes called structure adaptation. In structure adaptation, two methods are primarily used for the system (1)–(3). The first method is based on the extended Kalman filter (EKF) [8–12], and the second one is based on the standard Kalman filter and the Lainiotis partition theorem [1–3]. Note that the second method is often called the adaptive Kalman filtering (AKF). Both filters EKF and AKF are based on the Bayesian approach in which the unknown parameter θ is assumed to be random with a prior known probability $p(\theta)$. The EKF represents a suboptimal nonlinear filtering algorithm to estimate the composite state vector $[x_t\theta]^{\text{T}}$ that contains θ as its component. However, it is difficult to estimate the effect of nonlinear approximations made in the suboptimal realization of EKF [8–12].

The AKF separates the filtering process x_t from identification of the unknown parameter θ [1–5]. In this paper we are interested in such an AKF that constitutes a partitioning of the original nonlinear filter into the bank of much simpler N local Kalman filters where each local filter uses its own system model (1), (2) matched to each possible parameter value $\theta = \theta_i$, $i = 1, \ldots, N$. This AKF is also referred to multiple model adaptive estimation [13–18]. The overall estimate of state of this AKF is given by a weighted sum of local Kalman estimates, thus it can be implemented on a set of parallel processors due to its inherent parallel structure. However, the optimal AKF's weights represent the conditional probabilities of the specific parameter values $p(\theta_i \vert y_0^t)$ which depend on current observations y_0^t and it is rather difficult to implement the AKF in real-time for high-dimension of state vector and large number of local Kalman estimates (filters).

In this paper, discrete filtering of continuous-time linear systems with uncertainties is considered. We extend the well-known optimal discrete and continuous AKFs [1–5] to the mixed continuous–discrete linear systems with parametric uncertainties. But the main objective of the present paper is to give