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A surface charge simulation method based on advanced numerical integration

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ABSTRACT

Numerical models for computing low-frequency electromagnetic fields can contain spatial 2D finite elements, which are numerically most demanding due to problem of singularity. In this paper, an advanced time-harmonic quasistatic surface charge simulation method for computation of scalar electric potential and electric field intensity distribution is presented. Subparametric spatial 2D finite elements with an arbitrary number of nodes for description of surface charge density distribution are developed. The problem of singularity that occurs in the double 2D integration over these elements is solved using an originally developed advanced numerical integration based on 2D Gaussian quadrature. Self and mutual coefficients of spatial 2D finite element nodes are numerically computed and included in the system of linear equations for surface charge density distribution computation. The accuracy of the computer program, based on the presented model, is shown in the chosen numerical example with known analytical solution. Numerical model and advanced integration presented herein could be easily extended to non-homogeneous regions and multilayer problems using the image method.

1. Introduction

High voltage air-insulated switchyards, substations and power lines feature complex three-dimensional (3D) layout of metallic conductors. Large time-harmonic currents and charges cause low-frequency (50/60 Hz) electromagnetic fields, wherein electric field intensity in their vicinity can exceed reference values prescribed by ICNIRP guidelines [1].

In numerical models for computing low-frequency electromagnetic fields, the problem can be considered as quasistatic [2–5]. Quasistatic fields vary slowly with time and therefore, attenuation and phase shift of these fields can be neglected. Numerical algorithms for computing electromagnetic fields in electric power substations are 3D algorithms [5–9]. Numerical algorithms for computing overhead power line electromagnetic fields are mostly two-dimensional (2D) [10,11], although in a certain number of real cases more sophisticated 3D algorithms can be taken into account [12,13].

Applied methodologies for computing low-frequency electromagnetic fields range from the method of moments (MOM) [14], finite-difference time-domain method (FDTD) [15,16], finite element method (FEM) [17], boundary element method (BEM) [18], charge simulation method (CSM) [19,20], surface charge simulation method (SCSM) [21], to name a few. Theoretical basis of each method, its advantages and limitations in comparison with other solution techniques are well-known [22].

In a wider time-harmonic quasistatic surface charge method for computing low-frequency electromagnetic fields of power lines and substations [23], thin-wire cylindrical segments of active and passive conductors, conductive metallic spheres and spatial 2D finite elements are included and all of these components are capacitively coupled. The foundation of the developed method is application of finite element technique to an integral equation formulation in the frequency domain. In this paper, time-harmonic quasistatic charge simulation method [23] is reduced to surface charge simulation method, i.e. to use of spatial 2D subparametric finite elements. Spatial 2D finite elements are numerically most demanding due to problem of singularity which occurs in the double 2D integration when they are taken into consideration [24,25]. Therefore, in this paper, discussion is limited only to these elements. Hence, subparametric spatial 2D finite elements, with an arbitrary number of nodes for a description of surface charge density distribution, are developed. Expressions for self and mutual coefficients of these spatial 2D finite element nodes are derived using the Galerkin–Bubnov method, which gives symmetric system of linear equations. The problem of singularity that

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occurs in the double 2D integration over spatial 2D finite elements is successfully solved using originally developed advanced algorithm for double 2D numerical integration based on Gaussian quadrature. In the numerical example, due to simplicity, existence of analytical solutions and comparability, equipotential circular metal plate in a homogeneous unbounded dielectric medium (air) is chosen. It is therefore a good example to verify the accuracy of the algorithm developed in this paper, focusing on the accuracy of the advanced double 2D numerical integration over spatial 2D finite elements. Presented algorithm is developed for AC fields, but can be easily reduced to DC fields if phasors of all input data have phase angles equal to zero.

2. Subparametric spatial 2D finite elements

In the finite element terminology, some conductive passive parts of electrical substations and towers of overhead power lines can be described by spatial 2D finite elements. In Fig. 1, spatial 2D finite element with 9 nodes for geometry mapping and 25 nodes for numerical approximation of the surface charge density distribution is shown.

In this model, for geometry mapping of spatial 2D finite element, a quadrilateral reference 2D finite element with four, eight or nine nodes can be used. There are no limitations for the geometrical shape, which can be arbitrary, and only suitable number of nodes has to be chosen. In the Fig. 2, quadrilateral reference 2D finite element with eight nodes for geometry mapping is shown. Furthermore, the surface charge density distribution over the quadrilateral reference 2D finite element is described by linear combination of shape functions joined to the element nodes. Number of nodes is arbitrary and coefficients of the linear combination are potentials of element nodes. In the developed model, there are more nodes for approximation of the surface charge density distribution than nodes for geometry mapping, thus the spatial 2D finite elements are subparametric.

Mapping functions of the quadrilateral reference 2D finite element with four, eight or nine nodes are equal to product of 1D Lagrange polynomials or serendipity polynomials for each coordinate axis in the local coordinate system (u, v) [26,27]. Partial derivatives of these functions along local axes can be obtained easily. In the special case, quadrilateral reference 2D finite element can be mapped into a triangular finite element (Fig. 3).

Numerical approximation of the surface charge density distribution over a reference 2D finite element can be written as:

$$\bar{\sigma} = \sum_{q=1}^{NC} N_q \cdot \bar{\sigma}_q \tag{1}$$

where NC is a total number of the reference 2D finite element local nodes, N_q is the shape function joined to the q-th local node of the reference 2D finite element and $\bar{\sigma}_q$ is the phasor of the surface charge density of the q-th local node.

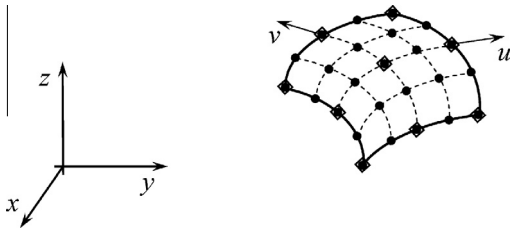

Fig. 1. Subparametric spatial 2D finite element in local coordinate system.

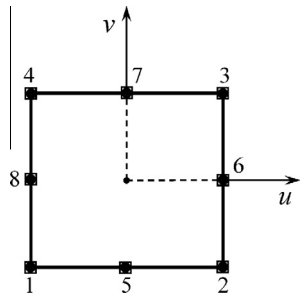

Fig. 2. Quadrilateral reference finite element with 8 nodes for geometry mapping.

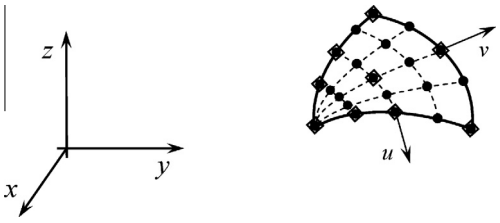

Fig. 3. Subparametric triangular spatial 2D finite element in local coordinate system.

Shape function joined to the q-th local node can be written as:

$$N_q = \left(\prod_{\substack{k=1 \\ k \neq cu}}^{NU} \frac{u - u_k}{u_{cu} - u_k} \right) \cdot \left(\prod_{\substack{k=1 \\ k \neq cv}}^{NV} \frac{v - v_k}{v_{cv} - v_k} \right); \quad q = 1, 2, \ldots, NC$$
$$q = (cv - 1) \cdot NU + cu; \quad cu = 1, \ldots, NU; \quad cv = 1, \ldots, NV \tag{2}$$

where NU is a total number of nodes along the local u-axis, NV is a total number of nodes along the local v-axis, $u \in [-1, 1]$ and $v \in [-1, 1]$ are local coordinates of the reference 2D finite element, whereas u_k and v_k denote local coordinates of the k-th node along u- and v-axes, respectively.

Hence, the total number of nodes of a reference 2D finite element for numerical approximation of the surface charge density distribution is given by:

$$NC = NU \cdot NV \tag{3}$$

Quadrilateral reference 2D finite element with 5 nodes along local axes u and v, i.e. with total 25 local nodes for numerical approximation of the surface charge density distribution is shown in Fig. 4.

Using the Galerkin–Bubnov method, which is a special case of weighted residual method, symmetric system of linear equations for computation of surface charge density of global nodes can be obtained from the following expression:

$$\iint_{S^i} (\bar{\varphi} - \bar{\Phi}^i) \cdot N_p^i \cdot dS^i = 0; \quad i = 1, \ldots, NP; \quad p = 1, \ldots, NC^i \tag{4}$$

where $\bar{\varphi}$ is a phasor of the computed value of the scalar electric potential, S^i is a surface of the i-th spatial 2D finite element, $\bar{\Phi}^i$ is a phasor of the prescribed potential on i-th spatial 2D finite element which is assumed to be constant, N_p^i is a shape function joined to the p-th local node of the i-th spatial 2D finite element, NP is a total number of the spatial 2D finite elements and NC^i is a total number of local nodes of the i-th spatial 2D finite element.

Continuity of the surface charge density at the spatial 2D finite element boundaries is not required. Consequently, local nodes of different spatial 2D finite elements cannot be joined to the same global node. For each subparametric spatial 2D finite element, global coordinates of local nodes for geometry mapping are input