Advances in Engineering Software 80 (2015) 25–32

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

Piecewise constant level set method for topology optimization of unilateral contact problems

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ARTICLE INFO

Article history:
Received 16 September 2014
Accepted 29 September 2014
Available online 25 October 2014

Keywords:
Unilateral contact problems
Topology optimization
Necessary optimality condition
Piecewise constant level set method
Augmented Lagrangian method
Uzawa method

ABSTRACT

The paper deals with the structural optimization of the elastic body in unilateral contact with a rigid foundation using the level set approach. A piecewise constant level set method is used to represent the evolution of interfaces rather than the standard method. The piecewise constant level set function takes distinct constant values in each subdomain of a whole design domain. Using a two-phase approximation the original structural optimization problem is reformulated as an equivalent constrained optimization problem in terms of the piecewise constant level set function. Necessary optimality condition is formulated. Finite difference and finite element methods are applied as the approximation methods. Numerical examples are provided and discussed.

1. Introduction

Contact phenomena among deformable bodies [1–3] abound in industry and everyday life, and play an important role in structural and/or mechanical systems. The complicated surface structure, physics and chemistry involved in contact processes make it necessary to model them with highly complex and nonlinear initial-boundary value problems. Considered in the paper, contact phenomenon with Tresca friction is governed by an elliptic variational inequality [4]. Structural optimization is classified as sizing, shape and topology optimization [5]. This paper is confined to consider topology optimization only. The aim of the topology optimization problem is to find such a material distribution inside the domain occupied by the elastic body in contact to minimize its normal contact stress along the boundary. The choice of such optimization criterion is motivated by applications. High normal contact stress results in surfaces fatigue, cracks or noise when the contacting bodies are moving. Topology perturbations of domain occupied by the body consist in nucleation or merging voids or weaker materials inside it [6,7] and are performed without any a priori assumption about the domain's topology. We do not consider a case of simultaneous boundary perturbations of domain as in classical shape optimization and topology perturbations [1]. These boundary perturbations consisting in moving the domain boundary in the direction of a suitable velocity field are performed under the assumption that the initial and final shape domains have the same topology [8].

A classical approach to solve topology optimization problems is based on the relaxed formulations and homogenization method (see [5,9,10]). The obtained optimal solution is a quasi-uniform distribution of composite materials rather than a classical design [9]. The density approach, also called the SIMP (Solid Isotropic Material with Penalization) method [9], is another currently used topology optimization method. It consists in the use of a fictitious isotropic material whose http://scholar.google.co.in/schhp?hl=en&as_sdt=0,5 elasticity tensor is assumed to be a function of penalized material density, represented by an exponent parameter. SIMP method has been used in [3] to solve numerically topology optimization problem for an elastic structure with unilateral boundary conditions. The other computational method often used to solve topology optimization problems is ESO (Evolutionary Structural Optimization) method (see [11]). Originally, this method is based on the concept of gradually removing unnecessary or inefficient material from a structure to achieve an optimal design. It combines biologically inspired evolutionary algorithms with SIMP or other approaches. The ESO method is easy to implement and link with existing finite element analysis software packages. These methods either tend to suffer from numerical instability [5] or require the advance calculus of the cost functional derivative. Besides SIMP or ESO approaches, in recent years the topological derivative method [6,12,13] has emerged as an attractive alternative to analyze and solve numerically topology optimization problems, especially of elastic structures, without employing the

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http://dx.doi.org/10.1016/j.advengsoft.2014.09.020
0965-9978/

homogenization approach. The topological derivative gives an indication on the sensitivity of the shape functional with respect to the nucleation of a small hole or a cavity or more generally a small defect in a geometrical domain around a given point. This concept of topological sensitivity analysis was introduced in the field of shape optimization in [13].

The level set based methods [14] have been proposed as a new type of structural optimization methods [7]. In structural optimization the level set method [7,15] is employed in numerical algorithms for tracking the evolution of the domain boundary on a fixed mesh and finding an optimal domain. This method is based on an implicit representation of the boundaries of the optimized structure. A level set model describes the boundary of the body as an isocontour of a scalar function of a higher dimensionality. While the shape of the structure may undergo major changes the level set function remains to be simple in its topology. These features make from the level set method robust procedure which makes easier to describe complex shape changes. The evolution of the boundary of the domain is governed by Hamilton–Jacobi equation. The speed vector field driving the propagation of the level set function is given by the Eulerian derivative of an appropriately defined cost functional with respect to the variations of the free boundary. The solution of this equation requires reinitialization procedure to ensure that it is as close as possible to the signed distance function to the interface. Moreover this approach requires regularization of non-differentiable Heaviside and Dirac functions. Applications of the level set methods in structural optimization can be found, among others, in [7,15–19]. In order to increase the numerical efficiency of the standard level set method different numerical improvements of this method employed for the numerical solution of the structural optimization problems have been proposed and numerically tested. These improvements or extensions of classical level set method include the employment of Fourier series expansion of the level set function, the extension of the velocity field in Hamilton–Jacobi equation from the domain boundary on the whole domain, the application of semi-Lagrange scheme for the solution of Hamilton–Jacobi equation or the employment of basis radial functions in the approximation of the level set function [18,19]. Since the standard level set method is not capable to generate voids the application of this method requires either the initial domain with distributed voids or the calculation of the topological derivative. In [5,20] a modified level set method combined with Tikhonov regularization of the cost functional has been used to solve two-dimensional and three-dimensional minimum mean compliance problems as well as eigenfrequencies maximization problems. In this approach Hamilton–Jacobi equation has been replaced by a reaction–diffusion equation ensuring the smoothness of the level set function.

Recently, a piecewise constant level set method as a variant of standard level set method has been proposed for the image segmentation [21], shape recovery [22] or elliptic inverse problems [23]. Piecewise constant level set function takes distinct constant values on each subdomain of the computational domain. For a computational domain divided into 2^N subdomains in standard level set approach is required 2^N level set functions to represent them. In piecewise constant level set method we can identify an arbitrary number of subdomains using only one discontinuous piecewise constant level set function. The interfaces between subdomains are represented implicitly by the discontinuity of a set of characteristic functions of the subdomains [21]. Comparing to the classical level set method, this method is free of the Hamilton–Jacobi equation and does not require the use of the signed distance function as the initial one. Piecewise constant level set method has been used in [24–26] and in [27] to solve numerically topological optimization problem in plane elasticity and for the Laplace equation in 2D domain, respectively. In [25] the method has been combined with MBO scheme to approximate the motion of an interface by its mean curvature. In [26] a binary level set method taking only values +1 or -1, as a special variant of a piecewise level set method has been used.

Due to unilateral boundary condition and/or friction as well as nonlinearity contact problems are difficult for analysis as well as for modeling and optimization. Topology optimization of unilateral boundary value problems has been considered in [1–3,6,16,18,28]. In [2,3] SIMP method has been used to solve optimization problem for frictionless contact. In [6] the topological derivative of the regularized cost functional has been used to find the solution to topology optimization problem for two-dimensional contact problem in elasticity. In [1,16,18] the level set method combined with the topological derivative method has been used to solve numerically topology optimization problems for two-dimensional unilateral boundary value problems. In [28] preliminary results have been reported on the application of the piecewise constant level set to solve structural optimization problem for elastic contact problem with Tresca friction.

In the paper the topology optimization problem for two-dimensional elastic contact problem with a given friction is formulated. This problem is solved using the piecewise constant level set method rather than classical level set method requiring the calculation of shape and/or topological derivatives of the cost functional as in [1]. In the paper using the ersatz material approach [7] this optimization problem is approximated in the computational domain by a two-phase optimization problem depending on weak phase material parameter. Using the piecewise constant level set method this two-phase optimization problem is reformulated as an equivalent constrained optimization problem in terms of the piecewise constant level set function only. The derivative of the cost functional is calculated and a necessary optimality condition is formulated. The piecewise constant level set function is approximated using the finite difference method. Finite element method is used to approximate the boundary value problem. This discretized constrained optimization problem is solved numerically using the Augmented Lagrangian method of Uzawa type. A gradient type algorithm is used. The proposed algorithm does not require to perform the complicated shape or topological sensitivity analysis or to implement re-initialization scheme. Some numerical implementation issues are discussed. The obtained numerical results are presented and discussed. The paper extends preliminary results contained in author's previous paper [28]. The extension concerns both the justification of the necessary optimality conditions formulation as well as analysis of the obtained numerical results.

2. Problem formulation

Consider deformations of an elastic body occupying two-dimensional domain Ω with the smooth boundary Γ (see Fig. 1). Assume $\Omega \subset D$ where D is a bounded smooth hold-all subset of R^2. Let $E \subset R^2$ and $D \subset R^2$ denote given bounded domains. So-called hold-all domain D is assumed to possess a piecewise smooth boundary. Domain Ω is assumed to belong to the set O_l defined as follows:

$$O_l = \{\Omega \subset R^2 : \ \Omega \text{ is open}, \ E \subset \Omega \subset D, \ {}^{\#}\Omega^c \leqslant l\}, \tag{1}$$

where ${}^{\#}\Omega^c$ denotes the number of connected components of the complement Ω^c of Ω with respect to D and $l \geqslant 1$ is a given integer. Moreover all perturbations $\delta\Omega$ of Ω are assumed to satisfy $\delta\Omega \in O_l$. The body is subject to body forces $f(x) = (f_1(x), f_2(x)), \ x \in \Omega$. Moreover, surface tractions $p(x) = (p_1(x), p_2(x)), \ x \in \Gamma$, are applied to a portion Γ_1 of the boundary Γ. We assume, that the body is clamped along the portion Γ_0 of the boundary Γ, and that the contact