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Recursive total instrumental-variable algorithm for solving over-determined normal equations and its applications

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Abstract

The well-known recursive least-squares (RLS) algorithms are directly dependent on the recursive estimation method for inversion of the auto-correlation matrix based on the matrix inverse lemma. On the basis of recursive estimation of the augmented cross-correlation matrix and the well-known power iteration, this paper establishes a recursive total instrumental-variable (RTIV) algorithm for tracking the total least-squares (TLS) solution of the normal equations in the over-determined instrumental-variable methods. Under the condition that the recursive estimation of the augmented cross-correlation matrix is asymptotically convergent, we show that the weight vector in the RTIV algorithm converges to the direction parallel to the singular vector associated with the smallest singular value of the augmented cross-correlation matrix. When the algorithm is applied in a parameter estimation problem, the converging weight vector corresponds to the TLS solution of the over-determined normal equations. Since the estimated parameters are optimal in the TLS sense, in the case where noise exists in both the input and output of the system or with the short sampling data, its noise rejection capability is superior to that of least-squares-based algorithms. The applicability and performance of the RTIV algorithm are demonstrated by computer simulations for adaptive IIR filtering and harmonic retrieval.

Keywords: Recursive least squares; Total instrumental variable; Algorithm; Convergence; IIR filtering; Harmonic retrieval

1. Introduction

Adaptive signal processing algorithms are widely applied in many areas because of their ability to adapt to the time varying and unknown environments. The recursive least-squares (RLS) algorithms for spectral estimation and system identification have been well developed and thoroughly analyzed, and the user is provided, at each sampling instant, with a set of parameters optimal in the least-squares (LS) sense [1–3]. If interference only exists in the output of the analyzed system, the RLS algorithms can give the optimal solution of an LS problem. However, if there is noise in both the input and output part of the analyzed system, the RLS algorithms can only lead to the sub-optimal solution of a signal processing problem [4]. Instead, the optimal solution can be obtained by the total least-squares (TLS)-based algorithms [5–9].

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The instrumental-variable (IV) approaches have been applied extensively in system identification [10,11] and array processing [12,13], and recently in blind signal separation [14] in an unknown noise environment, since they can efficiently suppress the colored noise. The over-determined IV methods, fast IV-lattice algorithms, and IV approaches for system identification and spectral estimation were described in [15–18]. The applications of the IV techniques in array processing were presented in [12,13,19]. More detailed reference lists can be found there.

Although the TLS problems were proposed in 1901 [20], Golub and Van Loan first studied their basic performance in 1980 [21]. The solutions of TLS problems have been extensively applied in signal processing (see [22–28]). In general, the solution of a TLS problem can be obtained by finding the singular vector associated with the smallest singular value of the matrix [21]. Since the number of multiplications involved in a singular value decomposition (SVD) of an $N \times N$ matrix is $O(N^3)$, applications of TLS problems are limited in practice, especially in real-time signal processing.

For adaptively computing the generalized eigenvector associated with the smallest eigenvalue of an auto-correlation matrix, a number of algorithms have been proposed in the context of Pisarenko spectral estimation [29]. These algorithms fall into the two broad classes. There are a large variety of algorithms of the first class, which require $O(N^2)$ multiplications per iteration. The representative algorithms include the inverse-power method [5], the conjugate-gradient method [6], and the LS-like method [7], all of which require $O(N^2)$ multiplications per eigenvector update. To solve the TLS problems in real-time signal processing with applications to adaptive FIR filtering, Davila proposed a fast recursive TLS (RTLS) algorithm based on a gradient search for the generalized Rayleigh quotient along the Kalman gain vector [42]. This algorithm can fast track the eigenvector associated with the smallest eigenvalue of the augmented auto-correlation matrix, since the Kalman gain vector can be fast estimated by the shift structure of the input data vector. Its computational complexity is $O(N)$ per iteration.

The second class involves stochastic type algorithms. Thompson [30] proposed an adaptive algorithm that is used to extract a single minor eigen-component and can be used to find the TLS solutions of adaptive filtering and on-line system identification problems. Other similar algorithms have also been reported by several authors in [30–32], all leading to an adaptive implementation of Pisarenko's harmonic retrieval estimator [5]. Yang and Kaveh [32] generalized Thompson's algorithm for estimating the complete minor components with the inflation procedure. However, Yang and Kaveh's algorithm needs division operation. Oja [33], Xu et al. [34], and Wang and Karhunen [35] proposed the similar algorithms, which avoid the division operation but require the assumption that the smallest eigenvalue is less than unity. Later on, Luo and Unbehauen [36] presented a minor subspace analysis (MSA) algorithm that attacks the above drawbacks. Gao et al. proposed a constrained anti-Hebbain learning algorithm used for solving TLS problems in adaptive FIR and IIR filtering [8]. The algorithm provides the simplest learning scheme for adaptive estimation under the TLS optimal criterion, but conditionally converges to the TLS solutions [37,38]. The convergence of these random adaptive algorithms was carefully analyzed in [39]. Moreover, a large amount of simulations has shown that these algorithms can work satisfactorily under the condition of the suitable learning rate and the appropriate initial values. In [9], a total least mean squares (TLMS) algorithm was developed from the point of view of total adaptive signal processing. In contrast, the TLMS algorithm has an equilibrium point under the persistent excitation condition [40]. The existing random algorithms require $O(N)$ multiplications per iteration, but have relatively slow convergence compared with the first class of adaptive algorithms. It should be pointed out that no algorithm in these two broad classes could be directly used to find the TLS solutions of the over-determined normal equations. Thus, it is necessary to extend the RTLS algorithms.

The main aim of this paper is to develop a recursive total instrumental-variable (RTIV) algorithm for finding the TLS solution to the over-determined normal equations, and to present its applications to adaptive IIR filtering and harmonic retrieval. The arithmetic operation complexity of the RTIV algorithm is $O(MN)$ per iteration, where M is the number of instrumental variables and N the dimension of the input vector.

2. TLS problem and its application in over-determined instrumental-variable methods

The TLS approach is an optimal technique that considers both the stimulation error and the response error. Here, the implication of TLS problems is illustrated by the solution to a set of conflict linear equations:

$$\mathbf{Ax} \approx \mathbf{b}, \tag{1}$$