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Filtering and prediction from uncertain observations with correlated signal and noise via mixture approximations

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Abstract

A recursive suboptimal filtering and prediction algorithm is designed to estimate Gaussian signals from uncertain observations, when the variables describing the uncertainty are independent, and the signal and observation white noise are correlated. The derivation is based on successive approximations of mixtures of normal distributions which, in turn, provide approximations of the conditional distributions of the signal given the observations. The proposed estimators, which are nonlinear functions of the observations, are then obtained as the expectation of these approximate conditional distributions. The algorithm does not require the whole knowledge of the state-space model generating the signal, but only covariance information of the signal and the observation noise, as well as the probability that the signal exists in the observed values.
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1. Introduction

The estimation problem in stochastic systems has been fairly reported in the literature under the assumption that the signal and the observation noise are correlated (see, for example, [1–3]), to treat several engineering and stochastic control problems where the signal–noise independence hypothesis is not realistic. In this paper, the estimation problem of Gaussian signals is considered using observations affected not only by an additive noise correlated with the signal but also by a multiplicative noise described by a sequence of Bernoulli random variables whose values—one or zero—indicate the presence or absence of the signal in the observation (*uncertain observations*). This kind of systems with uncertain observations are appropriate to model a large class of real situations with intermittent failures in the measurement mechanism, accidental loss of some observations, or inaccessibility of the data during certain time periods.

The least-squares (LS) optimal estimation problem in systems with uncertain observations is not easily treatable in general, due to the fact that the multiplicative noise perturbing the observations causes the joint distribution of the signal and the observations to be not Gaussian (even when the signal and the additive noise

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are both Gaussian processes); hence, the calculus of the optimal estimator requires an exponentially increasing memory and, consequently, it is not computationally practical. For this reason, the research on the estimation problem in these systems is focused on the search of suboptimal estimators of the signal. In this sense, assuming that the state-space model is completely known, the LS linear estimation problem has been widely investigated under different hypotheses on the processes involved and using different approaches (see [4–6] for the case where the signal and the observation additive noise are independent or [7,8] for a more general situation, where the additive noises of the state and the observation are correlated).

Besides the linear approximation, another possible approach to address the estimation problem in systems with uncertain observations is to approximate the conditional distributions which provide the optimal estimators; assuming knowledge of the state-space model, algorithms based on approximations of the conditional distribution by the empirical distribution of a set of random points (as, for example, particle filters frequently used in nonlinear systems) lead also to suboptimal estimators with, possibly, better performance than the linear ones (for references about this topic, see [9]). Moreover, it is important to indicate that, if the initial state and the additive noises are Gaussian, conditionally on the set of Bernoulli variables describing the uncertainty in the observations, these systems become linear and Gaussian; so, when the state-space model is known, systems with uncertain observations can be considered as a particular case of conditionally Gaussian linear state-space models, and the global sampling algorithm proposed in [10] can be applied.

Nevertheless, there exist many practical situations where the state-space model generating the signal is not available and the study of the estimation problem must be based on another kind of information. Under the assumption that the signal and the observation noise are independent, the LS linear estimation problem from uncertain observations using covariance information on the processes involved has been considered, for example, in [11,12]. More recently, this study has been completed in [13,14], by considering the one-stage prediction, filtering and smoothing problems from uncertain observations perturbed by an additive noise correlated with the signal.

The aim of this paper is to design a nonlinear algorithm for the filtering and prediction of uncertainly observed Gaussian signals correlated with the observation noise when the state-space model is unknown and only covariance information is available. The algorithm is based on approximated conditional distributions, but instead of approximations with empirical distributions, successive approximations of mixtures of Gaussian distributions by single Gaussian distributions are used. The proposed estimators are obtained as the expectation of these approximate conditional distributions. The algorithm only requires the knowledge of covariance functions of the processes involved, as well as the probability that the signal exists in the observed values. A numerical simulation example shows that, generally, the performance of the proposed nonlinear filter is significantly better than that of the linear one obtained in [14].

2. Observation model and problem formulation

Let z_k and y_k be $n \times 1$ vectors which describe the signal and the observation at time k, respectively. We assume that both vectors are related by the following observation equation:

$$y_k = \gamma_k z_k + v_k, \quad k \geqslant 1, \tag{1}$$

where v_k is the observation noise and γ_k is a Bernoulli random variable whose values—one or zero—indicate the presence or absence of signal in the observation y_k.

To treat the estimation problem of the signal from the observations given by Eq. (1), the following hypotheses are assumed:

(I) $\{(z_k^{\mathrm{T}}, v_k^{\mathrm{T}})^{\mathrm{T}}; k \geqslant 1\}$ is a Gaussian process with zero mean and its autocovariance function is expressed as

$$E\left[\begin{pmatrix} z_k \\ v_k \end{pmatrix} (z_s^{\mathrm{T}}, v_s^{\mathrm{T}})\right] = \begin{pmatrix} A_k B_s^{\mathrm{T}} & C_k D_s^{\mathrm{T}} \\ 0 & 0 \end{pmatrix}, \quad 1 \leqslant s < k,$$

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