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A bilevel integer programming method for blended composite structures

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ABSTRACT

This paper proposes a new approach for the design of a composite structure. This approach is formulated as an optimization problem where the weight of the structure is minimized such that a reserve factor is higher than a threshold. The thickness of each region of the structure is optimized together with its stacking sequence and the ply drop-offs. The novelty of this approach is that, unlike in common practice, the optimization problem is not simplified and split into two steps, one for finding the thicknesses and one for the stacking sequence. The optimization problem is solved without any simplification assumption. It is formulated as a bilevel integer programming and it uses the backtracking procedure to satisfy the blending and the manufacturing rules. Some numerical experiments are performed to show the efficiency of the proposed optimization method over complex cases which cannot be solved with the existing methods.

1. Introduction

In the recent years, composite materials have taken a growing importance in the aeronautical industry. Because they exhibit high performance properties and lead to a considerable weight reduction, they can be an alternative choice in the design of many aircraft parts. The design and manufacturing processes of a panel are based on a ply drop-off technique. If the panel is divided into regions (Fig. 1), each ply does not necessarily cover all the regions but some regions of it. As a consequence, the panel has a varying thickness over the surface of the panel which leads to a weight reduction. The fiber orientation in each ply can take one of the following values: $\{-45^\circ, 0^\circ, 45^\circ, 90^\circ\}$. The angle sequences of each region of the panel have to satisfy the blending and the manufacturing rules. The angle sequences must contain a fixed number of plies of each orientation, two consecutive angles cannot have a difference of 90°, there can be at most four consecutive identical orientations and the sequences must be symmetric. These constraints are called the design rules. The continuity of the plies between two adjacent regions of the panel is referred to as the manufacturing rules. The nature of such rules make the optimization problem a combinatorial one.

Many optimization methods based on genetic algorithms have been developed to address this specific problem. They differ by the technique which is used to satisfy the design/manufacturing rules. In [1–6], the manufacturing constraints are addressed using a penalty approach. In [7,8] a sub-laminate approach is used where regions sharing the same sub-laminates are grouped into one design variable. This method can guarantee the continuity of the plies in all the regions (blended structure).

In [9–12], the continuity of the plies (the blending) is satisfied by a guide-based design. It gives blended structures but it does not provide a lot of flexibility in the design of the panel: the stacking sequence of the thickest region imposes the stacking sequences of the all other regions. For a given stacking guide, only one sequence can be assigned to each region. To overcome this difficulty, a general definition of the blended sequences design is proposed in [13]. The blending constraint is taken into account with a penalty approach. However, severe stress concentration can be observed when the panel is not blended. The authors of this paper found that the penalty approach is not an efficient choice for satisfying the blending constraints. Another blending approach is the one described in [14,15], where the sequences of the regions are arranged into sets of plies which satisfy the blending principles. The approach in these two papers have the advantage of using the lamination parameters to compute the buckling instead of running expensive finite elements analysis.

In [16], the stacking sequence optimization problem is formulated as a linear integer programming problem where the orientation of each ply is modeled with four binary variables. The purpose of using these binary variables is to derive a mathematical expression of the manufacturing constraints. A linear expression of the buckling load is derived with respect to the stacking sequence in the case of a panel with one region. The paper studied the case

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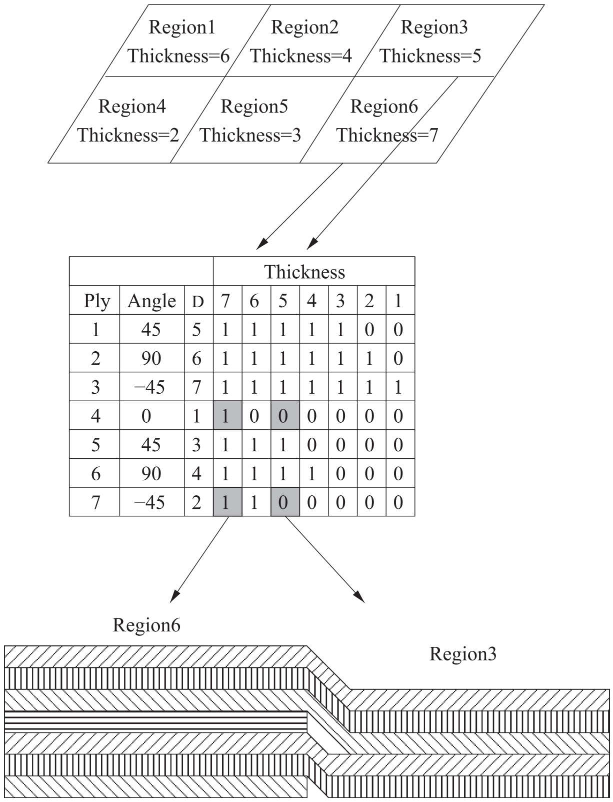

			Thickness						
Ply	Angle	D	7	6	5	4	3	2	1
1	45	5	1	1	1	1	1	0	0
2	90	6	1	1	1	1	1	1	0
3	−45	7	1	1	1	1	1	1	1
4	0	1	1	0	0	0	0	0	0
5	45	3	1	1	1	0	0	0	0
6	90	4	1	1	1	1	0	0	0
7	−45	2	1	1	0	0	0	0	0

Fig. 1. The blending principle and the ply drop-offs definition from a permutation vector *D*.

of a panel of one region with eight unknown plies to be optimized. This approach is limited to the case of such a panel. It cannot handle the general case of a panel with regions of different thicknesses, like in Fig. 1, because no linear expression for the buckling load can be derived. The same drawbacks have been found with the topology optimization approach proposed in [17–19]. They are able to optimize the buckling load with the manufacturing constraints but for a fixed blending scheme.

In [20], the authors have proposed a combinatorial method to optimize a buckling load with respect to the stacking sequence guide and the ply drop-offs, but the thicknesses were constant. This method has also been compared with other existing methods in [21] and in the case of single stacking sequences without blending. In this paper, the design of a composite panel is formulated as a bilevel integer programing where the weight of the panel is minimized subject to the buckling load higher than a safety threshold. The thicknesses of the regions are expressed in number of plies and they are updated together with the stacking sequence guide and the ply drop-off. The manufacturing and design rules are satisfied using the backtracking approach proposed in [20]. This approach does not use any approximation of it and can be generalized to a reserve factor of any type. The advantage of this approach over the classical one is discussed in the next section. The proposed algorithm in the paper only deals with the laminates of the structure and it cannot deal with other kind of composites. Therefore, the other parts of the structure remain constant during the optimization of the laminates. This is a limitation of the algorithm. If one is interested in optimizing the other parts, this must be after the optimization of the laminates and using other methods.

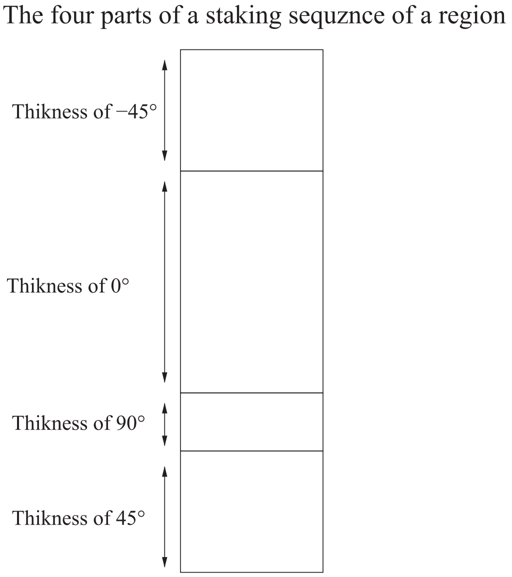

Fig. 2. The pattern of the stacking sequence in step one.

2. The two-step optimization

The full design of a composite panel consists in minimizing the weight of the panel and satisfying some buckling load constraints. This task is commonly divided into two steps, like in [14].

- *First step*: this step gives a global description of the panel without the details on the stacking sequences. The computation of the buckling loads is based on the following assumption. Four different orientations are considered and the stacking sequence of each region is divided into four parts, each part is associated to an orientation and has its own thickness (see Fig. 2). The design variables are the thicknesses per orientation and per region. They are continuous variables. Gradient based optimization methods over a finite elements code can be used to solve this problem.
- *Second step*: from the resulting thicknesses per orientation and per region of the preceding step, the number of plies of each orientation in each region is deduced. This step gives a detailed description of the stacking sequences by giving the arrangement of the plies in each region. Starting from the optimal configuration of the first step, the plies are permuted such that the panel comply with the blending and design rules and the buckling load is maximal.

This approach has the drawback that the sequences do not have a direct control over the thicknesses. If the second step gives a buckling load which is less than the safety threshold, then the thicknesses must be increased otherwise the overall optimization fails. If the second step gives a buckling load larger than the safety threshold, then the weight is not optimal and the thicknesses must be decreased.

Moreover the manufacturing constraints are hardly satisfied using the penalty method. The computational cost is high because two optimization problems are solved with two expensive methods.

The proposed bilevel approach overcomes these drawbacks by seeking simultaneously the thicknesses and the stacking sequences. At each iteration, the buckling load is computed using the current admissible stacking sequences. The thicknesses are updated according to this computed value. Then, the stacking sequences are updated based on the new thickness values.

Paper [20] studies the optimization problem of the second step. It considers a structure with different regions. Each region has its own thickness and it is defined by the number of plies. These