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Upper bound sequential linear programming mesh adaptation scheme for collapse analysis of masonry vaults

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ABSTRACT

The analysis of masonry double curvature structures by means of the kinematic theorem of limit analysis is traditionally the most diffused and straightforward method for an estimate of the load carrying capacity. However, the evaluation of the actual failure mechanism is not always trivial, especially for complex geometries and load conditions. Usually, the failure mechanism is simply hypothesized basing on previous experience, or – due to the complexity of the problem – FE rigid elements with interfaces are used. Both strategies may result in a wrong evaluation of the failure mechanism and hence, in the framework of the kinematic theorem of limit analysis, in an overestimation of the collapse load.

In this paper, a simple discontinuous upper bound limit analysis approach with sequential linear programming mesh adaptation to analyze masonry double curvature structures is presented. The discretization of the vault is performed with infinitely resistant triangular elements (curved elements basing on a quadratic interpolation), with plastic dissipation allowed only at the interfaces for possible in- and out-of-plane jumps of velocities. Masonry is substituted with a fictitious material exhibiting an orthotropic behavior, by means of consolidated homogenization strategies.

To progressively favor that the position of the interfaces coincide with the actual failure mechanism, an iterative mesh adaptation scheme based on sequential linear programming is proposed. Non-linear geometrical constraints on nodes positions are linearized with a first order Taylor expansion scheme, thus allowing to treat the NLP problem with consolidated LP routines.

The choice of inequalities constraints on elements nodes coordinates turns out to be crucial on the algorithm convergence. The model performs poorly for coarse and unstructured meshes (i.e. at the initial iteration), but converges to the actual solution after few iterations.

Several examples are treated, namely a straight circular and a skew parabolic arch, a cross vault and a dome. The results obtained at the final iteration fit well, for all the cases analyzed, previously presented numerical approaches.

1. Introduction

The analysis of masonry vaults up to collapse is a controversial issue that still is the object of investigation by specialized literature.

Far before the diffusion of computers, several graphical attempts for the study of the equilibrium of masonry domes were attempted by famous "fathers" of the mechanics, as for instance Bouguer (1734), Coulomb (1773), Bossut (1778) and Mascheroni (1785), who proposed simple mono-dimensional equilibrium equations, neglecting the role of circumferential actions. Anyway, what was clear from the beginning, was that non-linearity appears very early on curved masonry elements, even in presence of self-weight and with very low tensile stresses.

In this context, a considerable improvement in the analysis of spherical domes was achieved about 100 years later, when Levy (1888) proposed a graphical analysis aimed at finding the circle on which circumferential forces vanish. The history on the theories dealing with masonry vaults is long and fascinating and we refer the reader to the treatise by Benvenuto [1] for a comprehensive review.

Exception made for some particular cases, either where geometric and load symmetry may help in simplifying the problem or for single curvature structures (arches), and despite the considerable wide spreading of Finite Elements programs, it can be affirmed that, at present the models available to practitioners for a fast and reliable analysis of curved structural elements beyond

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the elastic limit are a few, see for instance the indications provided by Como [2], Heyman [3–5] and Huerta [6].

Limit analysis theorems associated with FEs, both in the static and kinematic version, are still the most effective and widespread procedure to estimate the collapse loads of one dimensional arches [7–12]. Indeed, limit analysis combines, on one hand, sufficient insight into collapse mechanisms, ultimate stress distributions – at least in critical sections – and load capacities, and on the other hand, simplicity to be cast into a practical computational tool. Given the difficulties in obtaining reliable experimental data for frictional materials, another appealing feature of limit analysis is the reduced number of necessary material parameters.

Similarly to arches, cupolas may be treated as well with 1D computerized approaches, but only under the quite restrictive condition of axi-symmetric loads [12–14].

Exception made for some special cases, the extension of automated approaches for complex geometries, general load conditions, reinforced arches and structures interacting with the infill still remains a challenging topic [15–19], despite experimentation in the field is putting at disposal a huge amount of experiences and evidences [20–22].

In absence of dedicated software, the most straightforward approach still remains the utilization of general purpose non-linear FEs, either already implemented in commercial codes [23,24] or non commercial but conceived for isotropic materials, as for instance concrete [18,19].

The author of this paper has been active on the implementation of FE limit analysis software specialized to the analysis of masonry structures from one decade. In this framework, some different approaches were proposed dealing with the prediction of the collapse loads and failure mechanisms of masonry vaults, taking into account some important distinctive aspects of the material, as orthotropy and geometry issues [25–30]. The models include (1) homogenized limit analyses by means of both plate and shell [30] and 3D elements [28,29].

An enhanced code [27] recently presented allows the possibility to model FRP reinforcement strips and steel tie rods, to quantitatively compare the situation before and after a rehabilitation intervention conducted with either innovative or traditional technology, thus implicitly selecting the most effective strategy for structural upgrading and refurbishment.

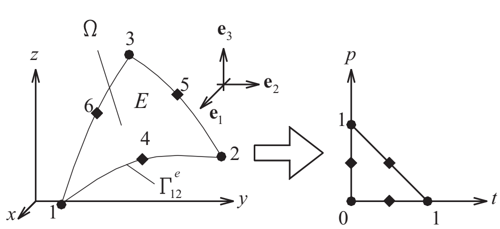

Fig. 1. Six-noded curved element and identification of Γ^e_{12} edge.

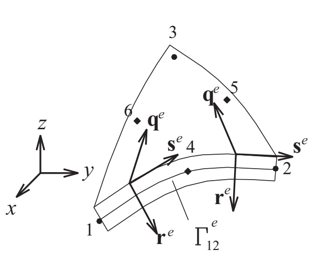

Fig. 2. Γ^e_{12} edge with thickness and $\mathbf{s}^e - \mathbf{q}^e - \mathbf{r}^e$ curved local frame of reference.

The approaches proposed base almost always on the upper bound theorem of classic limit analysis, i.e. where constitutive materials are assumed rigid-perfectly plastic with infinite ductility and the flow rule is associated.

From a technical point of view, the FE procedure bases on the original idea firstly proposed by Sloan and Kleeman [31], who presented a plane strain upper bound approach with triangular discretization and possible plastic dissipation on both continuum (triangles) and at the interfaces between adjoining elements. It has been widely shown, indeed, that such approach is extremely effective for cohesive frictional materials and therefore adapts well to masonry [32].

To deal with complex coupled problems where both flexural and membrane loads may play a crucial role in the formation of the failure mechanism, as in the case of masonry vaults, the utilization of triangular discretizations with dissipation on both elements and interfaces appears hardily applicable, both for the difficulties in determining homogenized failure surfaces to be used in a continuum schematization through a Reissner–Mindlin plate and shell model (a failure surface with eight independent variables, i.e. three internal membrane actions, three moments and two out-of-plane shears) and for the prohibitive number of variables to use in the classic framework of linear programming, even for small-scale problems.

Recent trends in limit analysis (see for instance [33–41]) demonstrated that the utilization of LP in solving the typical linear optimization problem associated to the upper and lower bound problems of limit analysis is less effective than the application of robust non-linear programming routines (NLP), with the considerable advantage that the linearization of the material strength domain is avoided. This allowed a further improvement in the numerical efficiency of FE limit analysis programs.

Another fundamental issue of limit analysis is that the classical lower and upper bound theorems allow a rigorous bracketing of the exact collapse load for a perfectly plastic structure. As a consequence, when such theorems are used in combination with the finite element method, the ability to obtain tight bracketing depends not only on the efficient solution of the arising optimization problem, but also on the effectiveness of the elements employed. Classic approaches aimed at improving the performance is to increase the "quality" of velocity (or stress) field interpolation inside elements, for instance using polynomial expansions with degree larger than one [42]. Basing on this idea, for example the so called free Galerkin approach and the p-FEM were used in [43–45], respectively.

However, such high order elements pose a particular difficulty when (strict) upper bound analyses must be performed, since the flow rule is required to hold throughout each element, whereas practically it can only be enforced on a finite number of points. To circumvent such a limitation, a constant strain element combined with discontinuities in the displacement field (see again Sloan and Kleeman [31]) was proposed in the past.

A quite simple and diffused classic alternative is to use remeshing [46,47], which relies into the introduction of new nodes and elements on those regions of the structure inside the processing zone. Remeshing basically requires (1) a rule to decide where to refine the mesh and (2) to establish in which way the mesh must be automatically refined. Whilst the procedure is very straightforward, it has the obvious drawback of increasing exponentially the computational effort needed after a few iterations, because the discretization is continuously refined where needed. In the framework of an upper bound approach of limit analysis, one of the rules – but not the unique – that may be adopted to identify the zones needing remeshing is represented by the identification of those elements where the plastic dissipation is large. In order not to generate distorted meshes in the new iteration, either a regular