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Sub-optimal all-sky detection of periodic gravitational waves

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Abstract

The design of a (roughly speaking *optimal*) GLRT-based detector for continuous gravitational waves (GWs) is a simple task and results in a bank-of-filter structure. Unfortunately, the computational burden of such a GLRT is unaffordable with the current computational powers: resorting to some kind of sub-optimal detectors seems unavoidable. We propose three new detection algorithms that can be implemented with a reasonable computational complexity, and analyze their behaviors both in terms of error probabilities and in terms of computational costs. With respect to these performance figures the improvements over the classical GLRT are impressive.

Keywords: Astrophysical signal processing; Detection; Gravitational waves; Pulsar; Sub-optimal detector

1. Introduction

The direct observation of gravitational waves (GWs) by interferometric antennas is a challenging task of modern physics, with strong implications for the understanding of the Universe around us and of the laws that govern it. Not surprisingly, the physical literature is very rich with contributions addressing the issue; the interested reader is referred to URL `xxx.lanl.gov` as a suitable entry point of such literature. However, the number of papers published in the signal-processing literature with direct or indirect reference to the GW detection problem is amazingly limited [1–10]. This tendency could perhaps change with the current availability of the first *reliable* data runs collected by the inteferometric GW antennas: analyzing these data requires rather sophisticated signal-processing approaches aimed at detecting the useful signal embedded in the noise, and at estimating the parameters of physical interest.

Several projects around the world are devoted to the construction of Earth-based gravitational antennas exploiting interferometric principles: LIGO (USA), VIRGO (Italy–France), GEO600 (Germany), TAMA (Japan), and the human and economic resources invested in these projects are large. All these efforts follow from the commonly accepted opinion according to which the direct observation of GWs is one of the most relevant scientific goals of the history of physics.

The GW antenna is basically a large-scale (hundreds of meters or a few kilometers of length)

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interferometer, which detects an impinging GW by exploiting the associated variation of the distance between two reference masses, as predicted by Einstein's general relativity theory [11]. The order of magnitude of the expected variations are of one part over something like 10^{21}. Accordingly, the antenna is a very sophisticated device whose realization involves many different technologies for implementing powerful and extremely clean lasers, very advanced seismic suspensions, and so on. Such a top-of-technology interferometer is affected by a number of noise sources which make the reliable observation of GWs a challenging task. The most relevant noises are of photonic, thermal and seismic origin, and their statistical properties have been investigated in-depth in the literature.

As to the source, we focus on the detection of a particular kind of GW, usually referred to as continuous or periodic GW, which from a signal-processing perspective is basically nothing but an amplitude/frequency-modulated (AM/FM) sine-wave. The continuous GW originates from the rotation of a stellar mass (e.g., neutron stars) with a nonzero quadrupole moment, which imprints a sinusoidal gravitational perturbation. The relative motion between the source and the Earth-based detector induces three phase modulation terms: one related to the daily Earth rotation about its axis, one to the orbital motion of the interferometer about the Earth–Moon barycenter with monthly period, and the latter, yearly, to the orbital motion about the Earth–Sun barycenter. Finally, the anisotropy of the antenna pattern induces an AM effect in the received signal. Instead of providing a long (and unavoidably incomplete) list of reference papers dealing with signal and noise models here, we refer to the gr–qc archive which can be found at the already quoted `xxx.lanl.gov`.

In complying with our belief that the detection approaches presented in this paper are better understood in a simplified setup, we take the liberty of considering the signal/noise models reduced to their basic essence. Specifically, as to the useful signal we refer to a monochromatic wave with a single sinusoidal modulating term due to daily Earth rotation, thus neglecting the other two phase modulations with larger periods. Also neglected are spin-down phenomena that may cause a drift of the fundamental frequency, as well as possible movements of the source; see for instance [12]. While the signal we consider is a simplified version of what is actually expected, it still entails the main features relevant to the detection problem, as pointed out in [13], which is one of the earlier reference on the subject with a signal-processing perspective; see also [4,5,14]. Along the same lines, even though each GW antenna is actually characterized by a specific (and non-flat) noise power spectral density, we shall refer to an idealized model: we assume additive, zero-mean, stationary, white Gaussian noise.[1] It should be stressed that deviations from Gaussianity have been reported, and time variations of the statistical properties of the noise process are also expected. Extensions of our results to more complicated scenarios (e.g., further modulation terms in the useful signal and/or colored or non-Gaussian noise) are left for future work.

A major concern about the direct observation of continuous GWs by interferometric antennas is related to the computational burden of the detector. It is necessary to design an all-sky and all-frequency detector, which means that the detection algorithm should account for unknown source location (all-sky), and unknown emission frequency (all-frequency). Then, the well-known detector structure based on a Generalized Likelihood Ratio Test (GLRT) requires the computation of exceedingly large FFTs. To give an idea of the computational burdens, let us say that the signal should be collected at least for several months, at a sampling frequency of some kilohertz, and the FFT of such a data set must be computed. As pointed out in [12,17], there is no hope to implement the GLRT with the currently available, and even with the near-future computational resources. The conventional detector being unrealizable, there is room for exploring alternative approaches aimed at mitigating the computational burden, while hopefully maintaining the final performances at an acceptable level. This is the goal of this paper, the remainder of which is organized as follows: Section 2 presents the signal model, and Section 3 deals with the GLRT design and performance assessment. In Section 4 alternative detection schemes are derived, analyzed, and compared with the GLRT. Section 5 contains conclusive remarks. Some technical derivations are presented in two appendices, to improve the paper's readability.

[1] Needless to say, should the noise be modeled as non-white, a whitening approach could be simply pursued [15,16].