Numerical modelling of consolidation processes under the water level elevation changes

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ABSTRACT

The paper presents a numerical model of coupled hydro-mechanical behaviour of soils. The micro-mechanics model is based on the effective stress concept which covers the theory of deformation of soils (soil skeleton) and other porous materials. The final set of equations is simplified and derived for the water flow in a porous media and the spatial discretization is performed by the finite element method. The model was implemented to the SIFEL software package and some numerical examples are presented.

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1. Introduction

Generally, soils consist of three components – grains (skeleton), liquid (water) and gas (water vapour and air). The total stress in a soil can be decomposed to the effective stress between grains, the pore water pressure and the pore gas pressure. The transport of water and heat together with the deformation of solid represent a non-linear coupled problem which can be described by three types of equations. There are constitutive equations (retention curves, material properties), transport equations (Fick's law and Darcy's law) and continuity equations. After discretization of driving equations using the finite element method (FEM), a system of non-symmetrical and non-linear algebraic equations is generally obtained, even if the deformation of the solid is linear elastic. On the other hand, a moisture transfer is a very slow process and therefore, in case of linear consolidation, no iteration is necessary within the particular time steps. However, experiments show that such description of consolidation by the linear elastic model for a porous medium along with the constant permeability is not too realistic. At least the permeability is mostly subjected to variations reflecting its dependence on the void ratio. Similarly, the observed plastic behaviour of soils can be described by the Cam-Clay model with the bilinear form of the normal consolidation line (NCL) which involves the effect of over-consolidation and structure strength on time dependent processes in soils.

2. Heat and moisture transfer in porous medium

In the past decades, a significant improvement in numerical modelling of coupled heat and moisture transport in porous materials has been attained. There are many material models in literature that allow for description of coupled heat and moisture transport. For example, there are phenomenological models based on diffusion [1–3]. These models are suitable for numerical simulations and modelling of building structures under common climatic conditions. On the other hand, complex micro-mechanical based models, e.g. Lewis and Schrefler's approach [4] and Tenchev's approach [5], using averaging techniques are applied namely to modelling of concrete structures under high temperature conditions and modelling of soil behaviour.

2.1. Lewis and Schrefler's approach of the coupled heat and moisture transfer

2.1.1. Principles of mass and heat transfer – retention curves

Porous materials have the capability of absorbing moisture from an air environment due to adsorption forces, attracting molecules of vapour to solid parts of the porous system, and due to the depression of water pressure because of tension over concave menisci of water filled capillaries. Moisture in materials can be therefore present as moist air, water and ice or in some intermediate state as adsorbed phase on the pore walls, respectively. Generally, it is not possible to distinguish between different aggregate states and therefore the water content is defined as the ratio of total moisture weight to dry weight of a material [6].

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The equilibrium between water content and its local environment is represented by a retention curve of the material which relates the moisture w and the relative humidity h of the surrounding air. An example of retention curves is depicted in Fig. 1.

In the transient region II (see Fig. 1), the relationship between the relative humidity, the water content (saturation) and the capillary pressure in pores [4] can be described by:

$$p^c = p^g - p^w, \tag{1}$$

where $p^w > 0$ is the pressure of the liquid phase (water).

The pressure of moist air, $p^g > 0$, in the pore system is usually considered as the pressure in a perfect mixture of two ideal gases – dry air, p^{ga}, and water vapour, p^{gw} and can be described by:

$$p^g = p^{ga} + p^{gw} = \left(\frac{\rho^{ga}}{M_a} + \frac{\rho^{gw}}{M_w}\right)TR = \frac{\rho^g}{M_g}TR. \tag{2}$$

In this relation, ρ^{ga}, ρ^{gw} and ρ^g stand for respective intrinsic phase densities, T is the absolute temperature, and R is the universal gas constant.

Identity (2) defining the molar mass of moist air, M_g, in terms of molar masses of individual constituents is known as Dalton's law. The capillary pressure is larger the smaller the capillary radius is. It is shown thermodynamically that the capillary pressure can be expressed unambiguously by the relative humidity h using the Kelvin-Laplace law

$$h = \frac{p^{gw}}{p^{gws}} = \exp\left(-\frac{p^c M_w}{\rho^w RT}\right). \tag{3}$$

In the Eq. (3), the water vapour saturation pressure p^{gws} is a function of the temperature only.

Volume fractions of liquid water (nS_w) and gas (nS_g) (moist air) in a porous medium with the porosity n are defined by the following identity

$$(1 - n)\rho^s w = nS_w\rho^w + nS_g\rho^g, \qquad S_w + S_g = 1, \tag{4}$$

where the volume moisture w [kg kg^{-1}] is related to the water content u [kg m^{-3}] by the formula

$$u = (1 - n)\rho^s w = nS\rho^w + n(1 - S)\rho^g = nS(\rho^w - \rho^g) + n\rho^g. \tag{5}$$

In the soil mechanics, moisture retention curves are mostly substituted by the material relationship [4]

$$p^c = p^c(S_w, \Theta), \tag{6}$$

which is obtained from laboratory tests, where $\Theta = T - T_0$ is the temperature difference above the reference value T_0.

Retention of enthalpy in materials having heat capacities is a term deliberately used to emphasise the similarity with the description of the moisture retention. The retention of enthalpy is simply described as
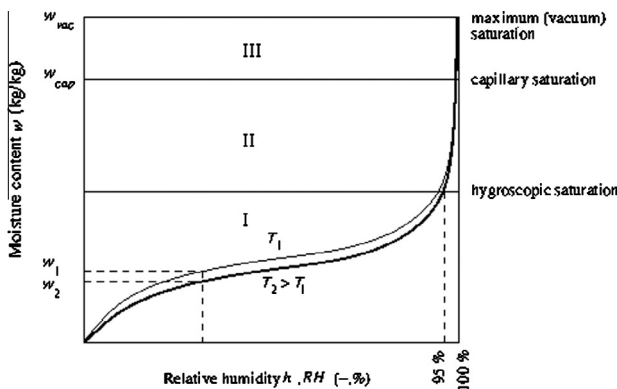

Fig. 1. Sorption isotherms.

$$H = H(T), \tag{7}$$

where H is the mass specific enthalpy [J kg^{-1}], T is the temperature [K].

It is not common to write the enthalpy in an absolute way as here. Instead, changes of enthalpy are described in a differential way, which leads to the definition of the specific heat capacity as the slope of the $H - T$ curve, i.e.

$$C_p = \left(\frac{\partial H}{\partial T}\right)_{p=\text{const.}}. \tag{8}$$

The heat capacity varies insignificantly with temperature but it is customary, however, to correct this term for the presence of fluid phases and to introduce the effective heat capacity as

$$(\rho C_p)_{\text{eff}} = \rho_s C_p^s + \rho_w C_p^w + \rho_g C_p^g. \tag{9}$$

Retention curves reflect material properties of porous media and their descriptions together with Eq. (8) are commonly known as state equations.

2.1.2. Moisture transfer – Darcy's and Fick's laws

A general approach to the coupled moisture and heat transfer in porous media is to express the vector of mass flux of moisture $\boldsymbol{J}$ and the heat flux vector $\boldsymbol{q}$ as a linear combination of a gradient of free (not chemically bound) pore-water content and of temperature as driving potentials. Off-diagonal phenomenological coefficients of a square (2 × 2) matrix describing this relation represent cross-effects – the Soret flux of moisture and the Dufour flux of heat (see, e.g. [7,8]). Since the gradients grad (w) and grad (T) are not generalised thermo-dynamical forces associated with fluxes $\boldsymbol{J}$ and $\boldsymbol{q}$, the coefficient matrix is generally non-symmetrical. In addition, the water content is not even a continuous potential. That is why a number of modified models have been developed (see, e.g., [3,7]) considering $\boldsymbol{w}$ as a function of temperature and pore water pressure and assuming that relevant functions are valid for the modified phenomenological coefficients.

If moisture convection is neglected, the liquid and gas (moist air) transport and the vapour diffusion taking place in the gas are remaining driving mechanisms.

Mass averaged relative velocities, $v^\alpha - v^s$, are expressed by the generalised form of Darcy's law [4]

$$nS_\alpha(\boldsymbol{v}^\alpha - \boldsymbol{v}^s) = \frac{k^{r\alpha}\boldsymbol{k}_{\text{sat}}}{\mu^\alpha}(-\text{grad }p^\alpha + \rho^\alpha\boldsymbol{g}), \tag{10}$$

where $\alpha = w$ for the liquid phase and $\alpha = g$ for the gaseous phase. Dimensionless relative permeabilities $k^{r\alpha} \in \langle 0, 1\rangle$ are functions of degree of saturation $k^{r\alpha} = k^{r\alpha}(S_w)$. $\boldsymbol{k}_{\text{sat}}$ [m^2] is the square (3 × 3) intrinsic permeability matrix and μ^α is the dynamic viscosity [kg m^{-1} s^{-1}]. Intrinsic mass densities ρ^α are related to volume averaged mass densities ρ_α through the relation

$$\rho_\alpha = nS_\alpha\rho^\alpha. \tag{11}$$

Diffusive-dispersive mass flux [kg m^{-2} s^{-1}] of the water vapour (gw) in the gas (g) is the second driving mechanism. It is governed by Fick's law

$$\boldsymbol{J}_g^{gw} = nS_g\rho^{gw}(\boldsymbol{v}^{gw} - \boldsymbol{v}^g) = -\rho^g\boldsymbol{D}_g^{gw}\text{grad}\left(\frac{\rho^{gw}}{\rho^g}\right), \tag{12}$$

where $\boldsymbol{D}_g^{gw}$ [m^2 s^{-1}] is the effective dispersion tensor. It can be shown [4] that

$$\boldsymbol{J}_g^{gw} = -\rho^g\frac{M_a M_w}{M_g^2}\boldsymbol{D}_g^{gw}\text{grad}\left(\frac{\rho^{gw}}{\rho^g}\right) = \rho^g\frac{M_a M_w}{M_g^2}\boldsymbol{D}_g^{ga}\text{grad}\left(\frac{\rho^{ga}}{\rho^g}\right)$$
$$= -\boldsymbol{J}_g^{ga}. \tag{13}$$