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Signal-to-noise ratio estimation using higher-order moments

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Abstract

We consider the problem of estimation of the signal-to-noise ratio (SNR) of an *unknown deterministic complex phase signal* in additive complex white Gaussian noise. The phase of the signal is *arbitrary and is not assumed to be known a priori* unlike many SNR estimation methods that assume phase synchronization. We show that the moments of the complex sequences exhibit useful mean-ergodicity properties enabling a "method-of-moments" (MoM)-SNR estimator. The Cramer–Rao bounds (CRBs) on the signal power, noise variance and logarithmic-SNR are derived. We conduct experiments to study the efficiency of the SNR estimator. We show that the estimator exhibits finite sample super-efficiency/inefficiency and asymptotic efficiency, depending on the choice of the parameters. At 0 dB SNR, the mean square error in log-SNR estimation is approximately 2 dB^2. The main feature of the MoM estimator is that it does not require the instantaneous phase/frequency of the signal, a priori. Infact, the SNR estimator can be used to track the instantaneous frequency (IF) of the phase signal. Using the adaptive pseudo-Wigner–Ville distribution technique, the IF estimation accuracy is the same as that obtained with perfect SNR knowledge and 8–10 dB better compared to the median-based SNR estimator.

Keywords: Signal-to-noise ratio; Phase signal; Moments; Mean square error; Cramer–Rao bound; Instantaneous frequency

1. Introduction

We address the problem of estimation of the signal-to-noise ratio (SNR) of a constant amplitude complex phase signal [1–3] s_n,[1] in additive complex white stationary Gaussian noise,[2] W_n. The noise is assumed to have zero-mean and unknown variance σ_w^2. The phase signal is deterministic with unknown amplitude A, phase ϕ_n, and is of the following form:

$$s_n = A\mathrm{e}^{\mathrm{j}\phi_n}. \tag{1}$$

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[1]The subscript is the discrete-time instant, assuming a normalized sampling period of unity.

[2]The signals in uppercase are random whereas those in lowercase are deterministic.

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We assume that A is real. If A is not real, the complex part of A can be absorbed by $e^{j\phi_n}$. This is the typical scenario of a time-varying signal in stationary noise. The noisy observations denoted by X_n, are given by

$$X_n = s_n + W_n, \quad 0 \leqslant n \leqslant N-1, \tag{2}$$

where N is the length of the random observation sequence. It is desired to estimate the signal-to-noise ratio, denoted by ξ and defined as $\xi = A^2/\sigma_w^2$.

The basic difference in the above model from those commonly used [4] is that s_n, *the signal of interest is deterministic but with a time-varying spectrum*. Also, many SNR estimation algorithms [4,5] assume that the phase is known/estimated (phase-synchronous SNR estimation). In the new technique, we do not assume a priori knowledge/estimate of the phase. Also, unlike many communication system applications, the phase signal is not constrained to have a constant frequency. We allow for arbitrary frequency variation which is not known a priori. This is a generalized signal definition and, is useful in many practical problems in digital communication systems [6,7], RADAR [8], SONAR, helicopter return signal analysis [9], aircraft flight parameter estimation [10], etc.

The organization of the paper is as follows: In Section 2, we state and prove the properties of the random sequence X_n that enable the *moments*-based SNR estimation. We derive the estimators for the signal power, noise variance and logarithmic SNR (Section 3.3) and show their statistical efficiency with respect to the CRB. We demonstrate, experimentally that the new estimator exhibits finite sample inefficiency/super-efficiency, depending on the choice of the parameters, A^2 and σ_w^2. However, it is asymptotically efficient. In Section 4, we compare the new estimator to the median-based SNR estimator. We consider the application to instantaneous frequency estimation (Section 4.2), and show that, in the presence of noise, the moments-SNR estimator improves the accuracy of the adaptive window pseudo-Wigner–Ville distribution based instantaneous frequency (IF) estimation technique significantly.

2. Properties of $|X_n|^2$ and $|X_n|^4$

We have $X_n = Ae^{j\phi_n} + W_n$, where W_n is independent and identically distributed (i.i.d) complex Gaussian distributed random noise of zero-mean and variance σ_w^2. The ensemble averages of the sequences, X_n and $|X_n|^k$ (k odd), $\mathscr{E}\{X_n\}$ and $\mathscr{E}\{|X_n|^k\}$ (k odd) are functions of time, i.e., they are non-stationary. However, the sequences $|X_n|^k$ (k even) are wide-sense stationary. Of particular interest are the sequences $|X_n|^2$ and $|X_n|^4$ which exhibit interesting properties as shown below:

(1) **The sequence $|X_n|^2$ is wide-sense stationary**. Consider the sequence $|X_n|^2$,

$$|X_n|^2 = A^2 + |W_n|^2 + Ae^{j\phi_n}W_n^* + Ae^{-j\phi_n}W_n. \tag{3}$$

Its expectation, denoted by $\mathscr{E}\{|X_n|^2\}$, is given by

$$\mathscr{E}\{|X_n|^2\} = A^2 + \sigma_w^2. \tag{4}$$

Therefore, the mean is constant. Now consider the autocorrelation, $\mathscr{E}\{|X_n|^2|X_m|^2\}$. Substituting for $|X_n|^2$ and $|X_m|^2$ in the above equation and making use of the properties that the noise is zero-mean, the real and imaginary parts of W_n are zero-mean, *i.i.d*, and each of variance $\sigma_w^2/2$, we get,[3]

$$\mathscr{E}\{|X_n|^2|X_m|^2\}_{n\neq m} = A^4 + 2A^2\sigma_w^2 + \sigma_w^4 \tag{5}$$

and

$$\mathscr{E}\{|X_n|^2|X_m|^2\}_{n=m} = A^4 + 4A^2\sigma_w^2 + 2\sigma_w^4. \tag{6}$$

Therefore, $\mathscr{E}\{|X_n|^2|X_m|^2\}$ is constant. Thus, the sequence $|X_n|^2$ is wide-sense stationary.

(2) **The sequence $|X_n|^2$ is mean-ergodic**. Consider the sample mean of $|X_n|^2$:

$$Y_N = \frac{1}{N}\sum_{n=0}^{N-1} |X_n|^2. \tag{7}$$

Its expectation is given by

$$\mathscr{E}\{Y_N\} = \frac{1}{N}\sum_{n=0}^{N-1} \mathscr{E}\{|X_n|^2\} = A^2 + \sigma_w^2. \tag{8}$$

[3] The derivation is given in Appendix 1.