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Optimization of laminate stacking sequence for minimizing weight and cost using elitist ant system optimization

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ABSTRACT

This paper presents the application of ant colony optimization (ACO) for the multi-objective optimization of hybrid laminates for obtaining minimum weight and cost. The investigated laminate is made of glass–epoxy and graphite–epoxy plies to combine the lightness and economical attributes of the first with the high-stiffness property of the second using a modified variation of ACO so called the elitist ant system (EAS) in order to make the tradeoff between the cost and weight as the objective functions. First natural frequency was considered as a constraint. The obtained results using the EAS method including the Pareto set, optimum stacking sequences, and the number of plies made of either glass or graphite fibers were compared with those using the genetic algorithm (GA) and any colony system (ACS) reported in literature. The comparisons confirm the advantage of hybridization and showed that the EAS algorithm outperformed the GA and ACS in terms of function's value and constraint accuracy.

1. Introduction

Laminated composites have been extensively used as structures in aerospace, defense, marine, automobile, and many other industries. This is because of the fact that they are generally lighter and stiffer than other structural materials. In contrast to isotropic materials, there are also some additional attributes in their design regarding the fiber orientations and stacking sequence which can be set in order to achieve the maximum efficiency.

In all applications, it is ideal to have the stiffest, lightest, and most economical structures. These three requirements normally act against each other and may come in compromise with the help of hybridization of composite laminates in which the high-stiffness material that is generally more expensive and heavier is used in the outer layers to provide enough rigidity and stiffness.

The material used in the inner layers should be light, bear less cost with low-stiffness. Deflection, stress, and natural frequencies are some supplementary aspects which have been investigated in hybrid laminates in a multi-objective optimization process. Maximizing the natural frequencies especially the fundamental one is critical importance in the design of laminates to decrease the risk of resonance caused by external excitations.

Material, thickness, and quantity of the surface and core layers as well as fiber orientations are the design variables in this process. However, in many engineering applications, it is reasonable to make use of standard layers with certain thicknesses and limited number of angles. Single-objective maximization of the fundamental frequency for laminated plates was given by [1–3] using continuous design variables.

The same design for cross-ply laminates was studied by Duffy and Adali [4] and for anisotropic laminates by Adali [5]. The minimum cost design of laminated plates undergoing free vibrations was investigated by Adali and Duffy [6]. Adali and Verijenko [7] investigated the optimum stacking sequence design of symmetric hybrid laminates undergoing free vibrations for fundamental frequency and frequency separation.

Spallino and Rizzo [8] presented the discrete optimization of laminated structures for the multi-objective optimization. Tahani et al. [9] optimized the fundamental frequency and cost in a multi-objective procedure using the genetic algorithm (GA), while Kolahan et al. [10] solved the same problem with the help of simulated annealing (SA).

Ant colony optimization (ACO) is a nature-inspired constructive based method which was first introduced by Dorigo and Gambardella [11] and so far has been extensively applied on various types of combinatorial problems such as Traveling Salesman Problem (TSP) [11], quadratic assignment, vehicle routing [12], and Job Shop Scheduling (JSP) [13].

There are many variations for ACO such as ant colony system (ACS) [11], max–min ant system (MMAS) [14], elitist ant system (EAS) [15], and rank-based ant system (AS_{rank}) [16]. The variations mostly differ in the global updating rule.

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Briefly, in ACS, all ants can leave pheromone on their trial [11], while in EAS, along with other ants, the global best solution (the best ant) deposits pheromone at every iteration [15]. In MMAS, only the global best ant allows to secrete pheromone on its trail with an extra constraint for pheromone value ($\tau_i \in [\tau_{\min}, \tau_{\max}]$) [14]. Accordingly, in AS_{rank}, all solutions are ranked according to their length. The amount of pheromone deposited is then weighted for each solution, such that solutions with shorter paths deposit more pheromone than the solutions with longer paths [16].

Camp et al. [17] studied the application of ACO for designing steel frames. Christodoulou [18] presented the optimal truss design using ACO. Kolahan et al. [19] also optimized a helical compression spring to achieve the minimum weight.

Grosset et al. [20] optimized the number of glass–epoxy and graphite–epoxy layer in order to obtain the minimum cost and weight subject to the first natural frequency using GA. Abachizadeh and Tahani [21] applied ACS on hybrid laminate composite for obtaining the minimum cost and weight and natural frequency as a constraint. The obtained results by ACS [21] were compared with the results gained by GA [20].

Recently, Abachizadeh and Tahani [22] studied the application of ACO for maximizing the fundamental frequency and minimizing the cost. They applied the ACO on two single-objective problems with 8-layerd graphite/epoxy laminates and 16-layerd glass/epoxy laminates with various aspect ratios and one multi-objective problem having 8, 16, and 28-layerd hybrid laminates with various aspect ratios. Their obtained results were compared to the results given by GA [9] and SA [10].

In this paper, weight and cost of symmetric balanced hybrid laminates were optimized considering the first natural frequency as the design constraint. The obtained results were compared with those using GA [20] and the ACS [21].

The remaining of this paper is organized as follows: In Section 2, a brief description for the analysis of free vibration and fundamental frequency in laminated plates are presented. In Sections 3 and 4, the concept of ant colony optimization is introduced and design problem is defined, respectively. In Section 5, a brief description of multi-objective optimization is presented. Section 6 deals with numerical results obtained using the EAS algorithm. Results, discussions, and comparisons for the performance of the EAS algorithm against GA [20] and ACS [21] are given in Section 7. Finally, conclusions are given in Section 8.

2. Analysis of fundamental frequency

Consider a simply supported symmetric hybrid laminated plate of length a, width b, and total thickness h in the x, y, and z directions, respectively. Each of the material layers has the thickness t and idealized as a homogeneous orthotropic material. The total thickness of the laminate is equal to $h = N \times t$, for which N is the total number of layers.

The hybrid laminate is made up of N_i inner and N_o outer layers so that $N = N_i + N_o$. The governing equation of motion within the classical laminated plate theory for the described symmetric laminate is given as follows [23]:

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 y^2} + 4D_{26}\frac{\partial^4}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4} = \rho h \frac{\partial^2 w}{\partial t^2} \quad (1)$$

where w is the deflection in the z direction, h is the total thickness, and ρ is the mass density averaged in the thickness direction which is given by:

$$\rho = h^{-1}\int_{-h/2}^{h/2} \rho^{(k)} dz = \frac{1}{N}\sum_{k=1}^{N} \rho^{(k)} \quad (2)$$

where $\rho^{(k)}$ denotes the mass density of material in the kth layer. The bending stiffnesses D_{ij} in Eq. (1) are defined as:

$$D_{ij} = \sum_{k=1}^{N}\int_{z_k}^{z_{k-1}} \overline{Q}_{ij}^{(k)} z^2 dz \quad (3)$$

where $\overline{Q}_{ij}^{(k)}$ is the transformed reduced stiffness of the kth layer which is given by:

$$\overline{Q}_{11} = Q_{11}\cdot\cos^4(\theta) + 2(Q_{12} + 2Q_{66})\cdot\sin^2(\theta)\cdot\cos^2(\theta) + Q_{22}\cdot\sin^4(\theta) \quad (4a)$$

$$\overline{Q}_{22} = Q_{11}\cdot\sin^4(\theta) + 2(Q_{12} + 2Q_{66})\cdot\sin^2(\theta)\cdot\cos^2(\theta) + Q_{22}\cdot\cos^4(\theta) \quad (4b)$$

$$\overline{Q}_{12} = (Q_{11} + Q_{22} - 2\cdot Q_{66})\cdot\sin^2(\theta)\cdot\cos^2(\theta) + 2(Q_{12} + Q_{12}\cdot(\sin^4(\theta) + \cos^4(\theta)) \quad (4c)$$

$$\overline{Q}_{16} = (Q_{11} - Q_{22} - 2\cdot Q_{66})\cdot\sin(\theta)\cdot\cos^3(\theta) + (Q_{12} - Q_{22} + 2\cdot Q_{66})\cdot\sin^3(\theta)\cdot\cos(\theta) \quad (4d)$$

$$\overline{Q}_{26} = (Q_{11} - Q_{22} - 2\cdot Q_{66})\cdot\sin^3(\theta)\cdot\cos(\theta) + (Q_{12} - Q_{22} + 2\cdot Q_{66})\cdot\sin(\theta)\cdot\cos^3(\theta) \quad (4e)$$

$$\overline{Q}_{66} = (Q_{11} + Q_{22} - 2\cdot Q_{12} - 2\cdot Q_{66})\cdot\sin^2(\theta)\cdot\cos^2(\theta) + Q_{66}(\sin^4(\theta) + \cos^4(\theta)) \quad (4f)$$

where Q_{ij} is the stiffness of composite along principal axes which is given as:

$$Q_{11} = \frac{E_1}{1 - \upsilon_{12}\cdot\upsilon_{21}} \quad (5a)$$

$$Q_{12} = Q_{21} = \frac{\upsilon_{21}\cdot E_1}{1 - \upsilon_{12}\cdot\upsilon_{21}} \quad (5b)$$

$$Q_{22} = \frac{E_2}{1 - \upsilon_{12}\upsilon_{21}} \quad (5c)$$

$$Q_{66} = G_{12} \quad (5d)$$

The boundary conditions for the simply supported plate are given as:

$$w = 0, M_x = 0 \Rightarrow x = 0, a \quad (6a)$$

$$w = 0, M_y = 0 \Rightarrow y = 0, b \quad (6b)$$

where the moment resultants are defined as:

$$(M_x, M_y) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y) dz \quad (7)$$

It is shown by Nemeth [24] that in buckling problems, the terms D_{16} and D_{26} which demonstrate the bending–twisting interactions in composite laminates can be safely neglected if the non-dimensional parameters satisfy the following constraints:

$$\gamma = D_{16}(D_{11}^3 D_{22})^{-1/4} \quad (8a)$$

$$\delta = D_{26}(D_{11} D_{22}^3)^{-1/4} \quad (8b)$$

$$\gamma \le 0.2 \quad (8c)$$

$$\delta \le 0.2 \quad (8d)$$

Because of the analogy between buckling and free vibration analysis, the same constraints are used to reduce the complexity of the problem. Taking into account the governing Eq. (1) and the boundary conditions in Eq. (6), a general form of solution for w in the natural vibration mode (m, n) is presented as:

$$w(x, y, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{i\omega_{mn} t} \quad (9)$$

where ω_{mn} is the natural frequency of the vibration mode (m, n) and $i = \sqrt{-1}$. Substituting Eq. (9) into Eq. (1) yields: