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Static analysis of functionally graded sandwich plates according to a hyperbolic theory considering Zig-Zag and warping effects

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ABSTRACT

In this paper, a variation of Murakami's Zig-Zag theory is proposed for the analysis of functionally graded plates. The new theory includes a hyperbolic sine term for the in-plane displacements expansion and accounts for through-the-thickness deformation, by considering a quadratic evolution of the transverse displacement with the thickness coordinate.

The governing equations and the boundary conditions are obtained by a generalization of Carrera's Unified Formulation, and further interpolated by collocation with radial basis functions.

Numerical examples on the static analysis of functionally graded sandwich plates demonstrate the accuracy of the present approach. The thickness stretching effect on such problems is studied.

1. Introduction

The strong difference of mechanical properties between faces and core in sandwich structures (or layered composites) introduces a discontinuity of the deformed core-faces planes at the interfaces. This is known as Zig-Zag (ZZ) effect. Such discontinuities make difficult the use of classical theories such as Kirchhoff [1] or Reissner–Mindlin [2,3] type theories (see the books by Zenkert [4], and Vinson [5] to trace accurate responses of sandwich structures). Two possibilities can be used to capture the ZZ effect (see the overviews by Burton and Noor [6], Noor et al. [7], Altenbach [8], Librescu and Hause [9], Vinson [10], and Demasi [11]): the so-called layer-wise models, and a Zig-Zag function (ZZF) in the framework of mixed multilayered plate theories. An historical review on ZZ theories has been provided by Carrera [12].

The first alternative can be computational expensive for laminates with large number of layers as the degrees-of-freedom increase as the number of layers increases. Considering the second alternative, Murakami [13] proposed a ZZF that is able to reproduce the slope discontinuity. Equivalent single layer models with only displacement unknowns can be developed on the basis of ZZF. A review of early developments on the application of ZZF has been provided in the review article by Carrera [14]. The advantages of analyze multilayered anisotropic plate and shells using the ZZF as well as the Finite Element implementation have been discussed by Carrera [15]. Further studies on the use of Murakami's Zig-Zag function (MZZF) have been documented in [15–17].

The use of alternative methods to the Finite Element Methods for the analysis of plates, such as the meshless methods based on radial basis functions (RBFs) is attractive due to the absence of a mesh and the ease of collocation methods. The use of radial basis function for the analysis of structures and materials has been previously studied by numerous authors [18–34].

Carrera's Unified Formulation (CUF) was proposed in [14,35,36] for laminated plates and shells and extended to functionally graded (FG) plates in [37–39]. The present formulation is a generalization of the original CUF in the sense that considers different displacement fields for in-plane and out-of-plane displacements.

In this paper the application of ZZF to bending analysis of thin and thick FG sandwich plates is studied. A new displacement theory is used, considering a quadratic variation of the transverse displacements (allowing for through-the-thickness deformations), and introducing a hyperbolic sine term in the in-plane displacement expansion. This can be seen as a variation of the original Murakami's ZZ displacement field. CUF is combined with RBFs for the static analysis: the principle of virtual displacements is used under CUF to obtain the governing equations and boundary equations and these are interpolated by collocation with RBFs.

The paper is organized as follows. The problem we are dealing with is introduced in Section 2. Then, the state-of-the-art review on the use of Zig-Zag functions and the displacement field of the

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Nomenclature

CUF	Carrera's Unified Formulation
FG	Functionally graded
FGM	Functionally graded material
FSDT	First-order shear deformation theory
MZZF	Murakami's Zig-Zag function
PDE	Partial differential equations
PVD	Principle of virtual displacements
RBF	Radial basis function
SSSS	Simply-supported
ZZ	Zig-Zag
ZZF	Zig-Zag function

present shear deformation theory is presented in Section 3. For the sake of completeness CUF and the radial basis functions collocation technique for the static analysis of FG plates are briefly reviewed in Sections 4 and 5, respectively. Numerical examples on the static analysis of simply supported functionally graded sandwich square plates are presented and discussed in Section 6. These include the computation of the displacements and stresses of sandwich plates with FGM in the core or in the skins, considering several material power-law exponents, side-to-thickness ratios and skin-core-skin ratios as well. Final conclusions are presented in Section 7.

2. Problem formulation

Consider a rectangular plate of plan-form dimensions a and b and uniform thickness h. The co-ordinate system is taken such that the x-y plane ($z = 0$) coincides with the midplane of the plate ($z \in [-h/2, h/2]$). The plate is subjected to a transverse mechanical load applied at the top of the plate.

Two different types of functionally graded sandwich plates are studied: sandwich plates with FG core and sandwich plates with FG skins.

In the sandwich plate with FG core the bottom skin is fully metal (isotropic) and the top skin is fully ceramic (isotropic as well). The core layer is graded from metal to ceramic so that there are no interfaces between core and skins, as illustrated in Fig. 1. The volume fraction of the ceramic phase in the core is obtained by adapting the typical polynomial material law as:

$$V_c = \left(0.5 + \frac{z_c}{h_c}\right)^p \tag{1}$$

where $z_c \in [h_1, h_2]$, $h_c = h_2 - h_1$ is the thickness of the core, and $p > 0$ is the power-law exponent that defines the gradation of material properties across the thickness direction as shown in Fig. 3 (left).

In sandwich plates with FG skins the core is fully ceramic (isotropic) and skins are composed of a functionally graded material across the thickness direction. The bottom skin varies from a metal-rich surface ($z = -h/2$) to a ceramic-rich surface while the top skin face varies from a ceramic-rich surface to a metal-rich surface ($z = h/2$), as illustrated in Fig. 2. There are no interfaces between core and skins. The volume fraction of the ceramic phase in the skins is obtained as:

$$\begin{aligned} V_c &= \left(\tfrac{z-h_0}{h_1-h_0}\right)^p, \quad z \in [-h/2, h_1] \\ V_c &= \left(\tfrac{z-h_3}{h_2-h_3}\right)^p, \quad z \in [h_2, h/2] \end{aligned} \tag{2}$$

where $p \geqslant 0$ is a scalar parameter that allows the user to define gradation of material properties across the thickness direction of the skins. The $p = 0$ case corresponds to the (isotropic) fully ceramic plate.

The sandwich plate with FG skins may be symmetric or non-symmetric about the mid-plane as we may vary the thickness of each face. Fig. 3 (right) shows a non-symmetric sandwich with volume fraction defined by the power-law (2) for various exponents p, in which top skin thickness is the same as the core thickness and the bottom skin thickness is twice the core thickness. Such thickness relation is denoted as 2-1-1. A bottom-core-top notation is

Fig. 1. Sandwich plate with FG core and isotropic skins.

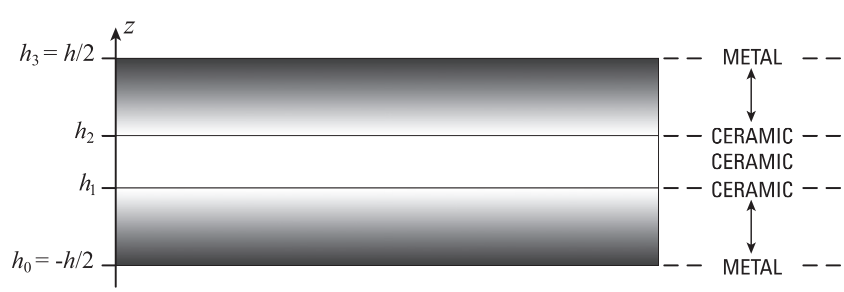

Fig. 2. Sandwich plate with isotropic core and FG skins.