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Eccentricity effects in the finite element modelling of composite beams

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ABSTRACT

When modelling composite or built up beams using finite element software, analysts find it often convenient to connect two standard Euler–Bernoulli beam elements at the nodes by using a rigid bar or use master–slave type kinematic constraints to express the degrees-of-freedoms of one of the members in terms of the other. However, this type of modelling leads to eccentricity related numerical errors and special solutions that avoid eccentricity related issues may not be available for a design engineer due to the limitations of the software. In this study, a simple correction technique is introduced in the application of master–slave type constraints. It is shown that the eccentricity related numerical errors in the stiffness matrix can be completely corrected by using extra fictitious elements and springs. The correction terms are obtained by using the exact homogenous solution of the composite beam problem as the interpolation functions which impose the zero-slip constraint between the two components in the point-wise sense. The effects of the eccentricity related errors are demonstrated in numerical examples.

1. Introduction

Composite beams and laminated plates that consist of components juxtaposed with a shear connection find widespread applications in structural engineering. A mathematical model for composite beams with flexible shear connectors was introduced by Newmark et al. [1], in which two Euler–Bernoulli beams are connected by assuming that vertical separation does not occur between the components. Subsequently, several displacement-based finite element formulations were developed based on Newmark's model, which include the works of Arizumi et al. [2], Daniels and Crisinel [3], Ranzi et al. [4,5] and Dall'Asta and Zona [6]. However, in many practical cases the interlayer connections are very stiff between the two components such that the interlayer slip is negligible. In such cases, displacement-based finite element formulations based on flexible shear connectors may suffer from the so-called slip-locking phenomenon because of the coupling between the displacement fields in the discretized form of Newmark's model [7]. Erkmen and Bradford [8] used the kinematic interpolatory strategy and assumed strain-mixed formulations to alleviate locking behaviour for stiff connections. An exact finite element formulation is also presented in [8]. A mesh free approach that eliminates slip locking was recently developed by Erkmen and Bradford [9].

Special solutions, however, are not always available for a design engineer in which case the modelling options are limited to standard conventional beam type finite elements. Therefore, a convenient practice in the modelling of composite beams especially for the cases with stiff connections is to connect the two conventional beam type finite element components by using a rigid bar to connect the end nodes of the two components or use master–slave type kinematic constraints to express the nodal degrees-of-freedom of one of the members in terms of the other. However, this type of modelling leads to eccentricity related numerical errors as initially shown by Gupta and Ma [10] in a composite cantilever beam problem. A similar type of error in multiple-point constraint applications for built-up plates and shells was pointed out by Crisfield [11]. Recently, Erkmen et al. [12] adopted variational multiscale approach to correct numerical errors in the applications of master–slave type constraints when forming composite beams.

In this study, a simple correction technique is introduced for the applications of master–slave type constraints to form composite beams and it is shown that the eccentricity related numerical errors in the stiffness matrix can be completely corrected by using extra fictitious elements and springs. The correction terms are obtained by selecting interpolation functions that do not violate the zero-slip constraint between the components in the point-wise sense. Since numerical errors are avoided the developed solution provides identical results to those that can be obtained using the elementary solution for composite sections, e.g. transformed area method in classical mechanics of solids books [13].

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2. Composite beam kinematics

2.1. Displacements and strains

The composite member is made up of a top and a bottom Euler–Bernoulli beam elements, which are referred to as members 1 and 2, respectively. Deformations of each component can be expressed in terms of the axial displacements w_i of the centroids and the vertical displacements v_i of the members, where $i = 1, 2$ throughout the document. Axial strain in each component of the composite beam can be determined by using the Euler–Bernoulli beam kinematics, hence in terms of the axial displacement gradients w_i' and the curvatures v_i'' due to bending, i.e.,

$$\varepsilon_i = w_i' - y_i v_i'' \tag{1}$$

where y_i refers to the vertical coordinates with respect to the associated beam component's centroid and $()' = \mathrm{d}()/\mathrm{d}z$.

3. Finite element formulation

A displacement based finite element formulation can be developed by employing the total potential energy functional, i.e.

$$\Pi = \frac{1}{2}\sum_{i=1}^{2}\int_L \int_{A_i} E_i \varepsilon_i^2 \mathrm{d}A\mathrm{d}z - \Pi_{ext} \tag{2}$$

where the first integral is the elastic bending energies of the two components, in which A_i is the cross-sectional area of the component and L is the span of the composite member, and Π_{ext} is the work done by the external forces. From Eqs. (1) and (2), the total potential energy functional in Eq. (2) can be written as

$$\Pi = \frac{1}{2}\sum_{i=1}^{2}\int_L \left(\langle w_i' \quad v_i'' \rangle \begin{bmatrix} E_iA_i & 0 \\ 0 & E_iI_i \end{bmatrix} \begin{Bmatrix} w_i' \\ v_i'' \end{Bmatrix} \right) \mathrm{d}z - \Pi_{ext} \tag{3}$$

where $I_i = \int_{A_i} y_i^2 \mathrm{d}A$ are the second moments of area of the beams with respect to their horizontal principal axes passing through the centroids of each cross-section.

3.1. Interpolation functions that satisfy kinematic constraints in the point-wise sense

When two beams are juxtaposed together, the common assumptions used in the practice are that there is no vertical separation between the two components and the cross-section remains planar after deformation. By using these kinematic conditions in the first variation of Eq. (3), the weak form of the equilibrium equations are obtained as

$$\int_L \left(\langle \delta w \quad \delta v \rangle \begin{bmatrix} \sum_{i=1}^{2} E_iA_i \mathscr{D}^2 & \sum_{i=1}^{2} E_iA_ih_i \mathscr{D}^3 \\ \sum_{i=1}^{2} E_iA_ih_i \mathscr{D}^3 & \sum_{i=1}^{2}\left(E_iA_ih_i^2 + E_iI_i\right)\mathscr{D}^4 \end{bmatrix} \begin{Bmatrix} w \\ v \end{Bmatrix} \right) \mathrm{d}z - \delta\Pi_{ext} - \delta\Pi_{Boun}\big|_0^L = 0 \tag{4}$$

in which $\mathscr{D} = \mathrm{d}/\mathrm{d}z$ is the differential operator, v is the vertical deflection of the composite beam based on an arbitrary common longitudinal axis z and, i.e. $v = v_i$ and h_i is the distance of the centroid of the component from the arbitrary common longitudinal axis. Integrations by parts have been used in obtaining Eq. (4). The boundary conditions are satisfied at $z = 0$ and $z = L$, i.e. $\delta\Pi_{Boun}|_0^L = 0$, thus the homogenous form of the field equations can be obtained from the first integral in Eq. (4) from which the solution for the displacement components of the arbitrary axis v and w can be obtained as

$$v(z) = \mathbf{N}(z)\mathbf{v}_N \tag{5}$$

and

$$w(z) = \mathbf{M}(z)\mathbf{w}_N - \frac{\sum_{i=1}^{2} E_iA_ih_i}{\sum_{i=1}^{2} E_iA_i}[\mathbf{N}'(z) - \mathbf{M}^*(z)]\mathbf{v}_N \tag{6}$$

where $\mathbf{v}_N^{\mathrm{T}} = \langle v(0) \quad v'(0) \quad v(L) \quad v'(L) \rangle$, $\mathbf{w}_N^{\mathrm{T}} = \langle w(0) \quad wL) \rangle$, $\mathbf{M}(z)$, $\mathbf{M}^*(z)$ and $\mathbf{N}(z)$ are

$$\mathbf{M}(z) = \langle (1 - z/L) \quad z/L \rangle \tag{7}$$

$$\mathbf{M}^*(z) = \langle 0 \quad (1 - z/L) \quad 0 \quad z/L \rangle \tag{8}$$

and

$$\mathbf{N}(z) = \left\langle \left(1 - \frac{3z^2}{L^2} + \frac{2z^3}{L^3}\right) \quad \left(z - \frac{2z^2}{L} + \frac{z^3}{L^2}\right) \quad \left(\frac{3z^2}{L^2} - \frac{2z^3}{L^3}\right) \quad \left(-\frac{z^2}{L} + \frac{z^3}{L^2}\right) \right\rangle \tag{9}$$

It should be noted that the axial displacement field in Eq. (6) is parabolic and this interpolation satisfies the kinematic constraints due to juxtaposition of the two components in the point-wise sense. The axial displacement field becomes linear only when the arbitrary axis coincides with the centroid of the whole composite section, i.e. $\sum_{i=1}^{2} E_iA_ih_i = 0$. From Eqs. (5) and (6), by employing the kinematic conditions again, i.e. $v = v_i$ and $w_i = w + h_i v'$ the displacement fields at the centroids of each component can be written as

$$v_i(z) = \mathbf{N}(z)\mathbf{v}_{Ni} \tag{10}$$

$$w_i(z) = \mathbf{M}(z)\mathbf{w}_{Ni} + \frac{E_mA_mh}{\sum_{k=1}^{2} E_kA_k}[\mathbf{N}'(z) - \mathbf{M}^*(z)]\mathbf{v}_{Ni} \tag{11}$$

where $\mathbf{w}_{Ni}^{\mathrm{T}} = \langle w_i(0) \quad w_i(L) \rangle$, $\mathbf{v}_{N\,i}^{\mathrm{T}} = \langle v_i(0) \quad v_i'(0) \quad v_i(L) \quad v_i'(L) \rangle$, h is the distance between the centroids of the beams, i.e. $h = h_m - h_i$ and $m = 1, 2$, $m \neq i$. In conventional finite element formulations the lack of second terms on the right hand side of Eq. (11) is the source of reduction in accuracy when two beams are juxtaposed with a rigid connection. The effects of these terms on the stiffness matrices will be shown in the following.

3.2. Stiffness matrix based on the exact interpolation field

From the first variation of the total potential energy functional in Eq. (3) and by using Eqs. (10) and (11), the weak form of the equilibrium equations can be written as

$$\sum_{i=1}^{2} \mathbf{K}_i \mathbf{U}_i = \sum_{i=1}^{2} \mathbf{F}_i \tag{12}$$

where $\mathbf{K}_i$ is the stiffness matrix of the member which can be written as

$$\mathbf{K}_i = \begin{bmatrix} \mathbf{K}_{11}^i & 0 \\ 0 & \mathbf{K}_{22}^i \end{bmatrix} \tag{13}$$

in which the sub-matrices are

$$\mathbf{K}_{11}^i = \begin{bmatrix} \frac{E_iA_i}{L} & -\frac{E_iA_i}{L} \\ -\frac{E_iA_i}{L} & \frac{E_iA_i}{L} \end{bmatrix} \tag{14}$$

and