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A novel scheme for large deflection analysis of suspended cables made of linear or nonlinear elastic materials

Najib Bouaanani *, Mohamed Ighouba

Department of Civil, Geological and Mining Engineering, Ecole Polytechnique de Montréal, Montréal, QC, Canada H3C 3A7

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ABSTRACT

This paper presents a new approach to investigate the static response of horizontal and inclined suspended cables with deformable cross-section, made of general linear or nonlinear elastic materials, and subjected to vertical concentrated and distributed loads. The proposed technique also includes large sag and extensibility effects, and is based on an original finite difference scheme combined to a nonlinear least squares numerical solution. The mathematical formulation is developed for various loading cases, and an innovative computational strategy is used to transform the resulting nonlinear system of equations into a scaled nonlinear least squares problem. The numerical scheme is programmed and its application illustrated through examples highlighting the effects of coupling between the tension in a cable and the deformation of its cross-section as well as the use of cables made of neo-Hookean materials. The results obtained are in excellent agreement with analytical solutions when available. The proposed technique can be easily programmed and constitutes a valuable tool for large deflection analysis of suspended cables made of nonlinear elastic materials.

1. Introduction

Cables are widely used as structural components and the investigation of their structural response has attracted numerous researchers for many centuries [1–3]. Most of these contributions accounted only for simplified loading cases and focused on validating the parabolic or catenary cable solutions amenable to hand calculations. With the advent of digital computers, advanced analytical and numerical techniques emerged as practical solutions to study cables under various loading cases, such as in [3–13] and more recently in [14–22]. Existing finite element software packages that solve cable problems generally use truss or beam elements including large displacement capabilities, i.e. geometrical nonlinearity. Such elements are highly effective solutions that avoid recourse to cumbersome cable modeling using 3D solid finite elements. However, the truss and beam formulations programmed into readily available finite element software are generally restricted to linear elastic Hookean materials, and do not allow for straightforward implementation of general constitutive nonlinear material models to account for hyper-elastic or rubber-like materials. Furthermore, these classical formulations do not account for coupling between the tension in a cable and the deformation of its cross-section which is assumed to remain rigid as the loads are applied. In this paper, we propose alternative finite difference solutions that waive these restricting assumptions. Finite difference modeling of cables was indeed shown very effective in many cable applications such as transmission lines, marine cables and cable-supported bridges [23–32]. Such studies showed that finite difference schemes can be easily programmed to yield robust numerical solutions and that their use is particularly justified when discretized nondimensional equations are to be solved systematically for problem parameters varying over a wide range. However, the available finite difference formulations for cables also employ a simplifying rigid cross-section kinematic assumption and are limited to linear Hookean materials.

The objective of this work is to develop an original and practical finite difference scheme to investigate the static response of horizontal and inclined suspended cables with deformable cross-section, made of general linear or nonlinear elastic materials, and subjected to most common loads of gravitational type, generally originating from self-weight, ice accumulation or various attachments. The proposed formulation also includes large sag and extensibility effects that were shown sufficiently important to include in the analysis of cables when large spans and/or significant loads are involved such as for applications described in [33–38].

* Corresponding author.
E-mail address: najib.bouaanani@polymtl.ca (N. Bouaanani).

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2. Mathematical and numerical formulations

2.1. Cable static response under self-weight

2.1.1. Mathematical formulation

In this section, we derive the Cartesian equations expressing the static profile of a suspended cable hanging under its self-weight as illustrated in Fig. 1. Considering a Cartesian system of axes (x,y,z), the cable is assumed to deflect within the plane (x,z). We note (x_A,y_A,z_A) and (x_B,y_B,z_B) the coordinates of the two cable supports A and B, respectively. To alleviate the notation, we may assume without loss of generality that support A coincides with the origin of axes, i.e. $x_A = y_A = z_A = 0$. The chord connecting the supports A and B makes an angle θ with the x-axis. We note $\widetilde{S}$ the unstrained arc-length of the cable, and $\widetilde{m}$g its weight per unit unstrained arc-length with g representing the gravity constant. The Lagrangian coordinate of a point of the cable in its unstrained configuration is denoted by $\widetilde{s}$. The static strained geometrical configuration is obtained when the cable deforms under self-weight. We note $\widehat{S}$ the strained arc-length of the cable, $\widehat{m}$g the weight of the cable per unit strained arc-length and $\widehat{s}$ the Lagrangian coordinate of a point of the cable in the strained configuration.

Fig. 1a illustrates the strained geometrical configuration of the suspended cable and the forces applied to an elementary segment of the cable with infinitesimal arc-length $\mathrm{d}\widehat{s}$. Let $\widehat{F}$ denote the tension force at a point with Lagrangian coordinate $\widehat{s}$ and Cartesian coordinate $\widehat{x}$. At coordinates $\widehat{s}+\mathrm{d}\widehat{s}$ and $x + \mathrm{d}x$, the tension force is $\widehat{F}+\mathrm{d}\widehat{F}$. The horizontal and vertical projections of the cable tensions $\widehat{F}$ and $\widehat{F}+\mathrm{d}\widehat{F}$ are designated by $\widehat{H},\widehat{V},\widehat{H}+\mathrm{d}\widehat{H}$ and $\widehat{V}+\mathrm{d}\widehat{V}$ as illustrated in Fig. 1a. The equilibrium of the elementary segment yields

$$\mathrm{d}\widehat{H} = 0 \tag{1}$$

$$\frac{\mathrm{d}\widehat{V}}{\mathrm{d}\widehat{x}} = -\widehat{m}\mathrm{g}\sqrt{1+\left(\frac{\mathrm{d}\widehat{z}}{\mathrm{d}\widehat{x}}\right)^2} \tag{2}$$

$$\widehat{H}\frac{\mathrm{d}\widehat{z}}{\mathrm{d}\widehat{x}} - \widehat{V} = 0 \tag{3}$$

Eq. (1) shows that horizontal tension $\widehat{H}$ is constant along cable arc-length, and the last two relations yield

$$\widehat{H}\widehat{z}'' + \widehat{m}\mathrm{g}\sqrt{1+(\widehat{z}')^2} = 0 \tag{4}$$

where the notations $\widehat{z}' = \mathrm{d}\widehat{z}/\mathrm{d}\widehat{x}$ and $\widehat{z}'' = \mathrm{d}^2\widehat{z}/\mathrm{d}\widehat{x}^2$ are used to alleviate the text. The tension force $\widehat{F}$ can be decomposed as

$$\widehat{F} = \widehat{H}\cos\widehat{\phi} + \widehat{V}\sin\widehat{\phi} = \widehat{H}\frac{\mathrm{d}\widehat{x}}{\mathrm{d}\widehat{s}} + \widehat{V}\frac{\mathrm{d}\widehat{z}}{\mathrm{d}\widehat{s}} \tag{5}$$

in which $\widehat{\phi}$ is the angle between the tangent to the cable profile and a horizontal axis as illustrated in Fig. 1a. Using Eqs. (1), (2), (3) and (5), we obtain

$$\widehat{F} = \widehat{H}\frac{\mathrm{d}\widehat{x}}{\mathrm{d}\widehat{s}} + \left(\widehat{H}\frac{\mathrm{d}\widehat{z}}{\mathrm{d}\widehat{x}}\right)\frac{\mathrm{d}\widehat{z}}{\mathrm{d}\widehat{s}} = \widehat{H}\sqrt{1+(\widehat{z}')^2} \tag{6}$$

The masses $\widetilde{m}$ and $\widehat{m}$ are distributed per unit unstrained and strained arc-lengths, respectively, and are related by

$$\widehat{m} = \widetilde{m}\frac{\mathrm{d}\widetilde{s}}{\mathrm{d}\widehat{s}} \tag{7}$$

and axial deformation can be characterized along cable arc-length by

$$\frac{\mathrm{d}\widehat{s}-\mathrm{d}\widetilde{s}}{\mathrm{d}\widetilde{s}} = \mathcal{C}(\widehat{\tau}) \tag{8}$$

in which

$$\widehat{\tau} = \frac{\widehat{F}}{EA} = \frac{\widehat{H}}{EA}\sqrt{1+(\widehat{z}')^2} \tag{9}$$

and where A is the area of the cable cross-section, E is the modulus of elasticity and $\mathcal{C}$ is a general constitutive function characterizing cable axial deformation. For example, $\mathcal{C}$ can be expressed in the simple case of a Hookean material as

$$\mathcal{C}(\widehat{\tau}) = \widehat{\tau} \tag{10}$$

More complex expressions of $\mathcal{C}$ will be investigated later in Section 3 of this paper.

Using Eq. (8), Eq. (7) becomes

$$\widehat{m} = \frac{\widetilde{m}}{1+\mathcal{C}(\widehat{\tau})} \tag{11}$$

Substituting Eqs. (11) and (6) into Eq. (4) yields the nonlinear differential equation governing the static profile of the cable including extensibility and large sag effects

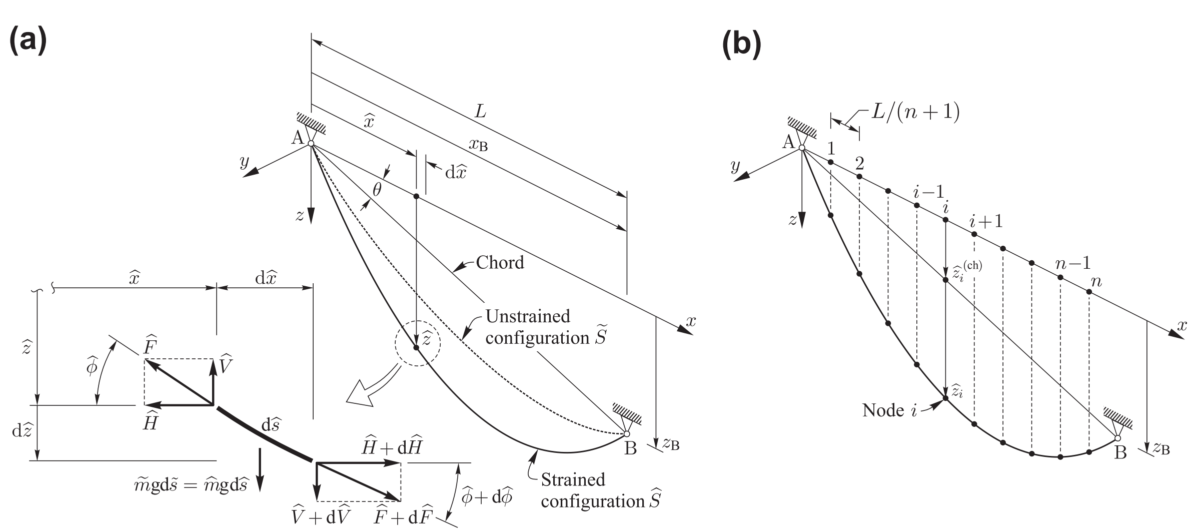

Fig. 1. Static response of a suspended cable: (a) Unstrained and strained geometrical configurations and equilibrium of an elementary segment of the cable; (b) Finite difference mesh.