Advances in Engineering Software 42 (2011) 305–315

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

Computer modelling of wire strands and ropes Part I: Theory and computer implementation

E. Stanova [a], G. Fedorko [b], M. Fabian [c], S. Kmet [a,*]

[a] Faculty of Civil Engineering, Technical University of Kosice, Vysokoskolska 4, 042 00 Kosice, Slovak Republic
[b] Faculty of Mining, Ecology, Process Control and Geotechnology, Technical University of Kosice, Park Komenskeho 14, 042 00 Kosice, Slovak Republic
[c] Faculty of Mechanical Engineering, Technical University of Kosice, Letna 9, 042 00 Kosice, Slovak Republic

ARTICLE INFO

Article history:
Received 17 February 2011
Accepted 18 February 2011
Available online 26 March 2011

Keywords:
Single-lay wire strands
Double-lay wire ropes
Single and double helixes
Geometric parametric equations
Computer modelling
Rope model

ABSTRACT

In this paper the mathematical geometric models of the single-lay wire strands and double-lay wire ropes with defined initial parameters are presented. The present geometric models fully consider the single-helix configuration of individual wires in the strand and the double-helix configuration of individual wires within the wound strands of the ropes. The mathematical representation of the single and double helixes is in form of parametric equations with variable input parameters which determine the centreline of an arbitrary circular wire of the right hand lay and left hand lay strands and ropes of the Lang lay and regular lay construction. The concrete forms of the parametric equations are derived and presented. The application of the derived geometric analytical model is illustrated by numerical examples. Techniques for the implementation of the derived mathematical models in CATIA V5 software and procedures for the generation of the rope model are briefly presented. Correctness of the derived parametric equations and a performance of the generated rope model are controlled by visualizations. The application of the derived mathematical model and the development of a finite element model for the numerical simulation of the multi-layered strand under tension tests are treated in the second part of the paper [1].

1. Introduction

Wire strands and ropes are used in a variety of engineering applications due to their high strength-to-weight ratio and very efficient use of the material. These characteristics are of particular significance in the design of lightweight suspended structures and large-span cable-stayed bridges.

Computer-aided design together with the finite element method create powerful sophisticated tools for the modelling and analysis of wire strands and ropes. These ropes often have highly complex constructions that require structural analysis beyond simple, idealized mathematical models and simplified computational approaches [2]. In the advanced design and analysis of wire strands and ropes three fundamental phases can be identified: creation of the geometric model, generation of the engineering cable model using the CAD system and application of the finite element method for an analysis under the required loading.

The geometric model plays key part in almost all further phases of the design and analysis process. The three dimensional shape of the single and double helixes defined by the mathematical geometric equations must be converted into an engineering strand and rope model and all of their constructional aspects (the single-helix configuration of individual wires in the strands and the double-helix configuration of individual wires within the wound strands of the ropes) must be specified at this phase.

Several authors have developed geometrical and analytical models of strands and ropes to predict their behaviour under various loads. The wire strands and ropes are treated either as a discrete set of concentric orthotropic cylinders (the individual layer of wires is replaced by an equivalent cylindrical orthotropic sheet) or as a configuration of helically curved rods, with different assumptions about the cable geometry or the inter-wire contacts, according to the authors. Utting and Jones' analysis [3,4] based on the classical twisted rod theories for the behaviour of helical laid wires takes the contact deformation and friction effects into account whereas Costello's approaches neglect them [5]. The orthotropic sheet model was first applied to cable modelling by Hobbs and Raoof [6] and then extended by Raoof and his associates over two decades. Velinsky [7] presented the closed-form analysis for elastic deformations of multi-layered strands and the design of wire ropes [8]. Lee [9] presented the geometrical analysis applicable to any rope with axisymmetric strands. He derived the Cartesian coordinate equations, which describe the helix geometry of wire within a rope. Through the application of differential geometry and the use of engineering

* Corresponding author. Tel.: +421 556022121.
E-mail address: stanislav.kmet@tuke.sk (S. Kmet).

0965-9978/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.advengsoft.2011.02.008

drawing development approach, problems associated with the three-dimensional helix geometry of wire rope could be solved. The derived geometric equations were used for an analysis of the geometrical properties of cables. Hobbs and Nabijou [10] studied the changes in curvature in single and double helixes as they are bent into circular arcs. This analysis was applied to wire ropes to examine the bending strains in the wires of a frictionless rope as it is bent over a sheave. Jolicoeur and Cardou [11] proposed semi-continuous mathematical model for the analysis of multilayered wire strands under bending, tensile and torsion loads. Each layer of a strand is mathematically represented by an orthotropic cylinder whose mechanical properties are chosen to match the behaviour of its corresponding layer of wires. Raoof and Kraincanic [12] presented the model for the theoretical analysis of a large-diameter wire rope, using results from a derived orthotropic sheet model, for analyzing the behaviour of its constituent helical strands. Knapp et al. [13] developed the CableCAD software code for the geometric modelling and finite element analysis of cables. Elata et al. [14] presented a new model for simulating the mechanical response of a wire rope with an independent wire rope core. In contrast with previous models that consider the effective response of wound strands, the present model fully considers the double-helix configuration of individual wires within the wound strand. This enables to directly relate the wire level stress to the overall load applied at the rope level. The double-helix geometry is modelled with the parametric equations because of its complex nature. A review of previous studies on the geometric modelling and analysis of steel and synthetic cables can be found in [15,16]. Synthetic fibre ropes are characterized by a very complex architecture and hierarchical structure. Leech et al. [17] presented a more complex analysis of fibre ropes and included it in the commercial software Fiber Rope Modeller (FRM). Usabiaga and Pagalday [18] derived the parametric equations of the double helical wires for the undeformed configuration of the rope.

In this paper, the improved mathematical models for geometric modelling of wire strands and ropes are derived and their implementation to the computer-aided design software CATIA V5 [19] is described. The mathematical model developed is able to generate both single- or multi-layer strands' and multi-strand ropes' geometries by the computer. The present geometric models fully consider the single-helix configuration of individual wires in the strand and the double-helix configuration of individual wires within the wound strands of the ropes. The mathematical representation of the single and double helixes is in form of parametric equations with variable input parameters which determine the axis of an arbitrary circular wire of the right hand lay and left hand lay strands and ropes of the Lang lay and regular lay construction. The concrete forms of the parametric equations are derived and presented. The application of the derived geometric analytical model is illustrated by numerical examples. Techniques for the implementation of the derived mathematical model in CATIA V5 software and procedures for the creation of the rope model are briefly presented. Correctness of the derived parametric equations and a performance of the generated rope model are controlled by visualizations. The application of the derived mathematical model and the development of a finite element model for the numerical simulation of the multi-layered strand under tension tests are treated in the second part of the paper [1].

2. Basic assumptions and problem formulation

The cables considered in this paper are a strand made of one or more layers of circular wires helically laid over a central circular straight core wire and a rope made of one or more layers of strands helically laid around a core. Let us specify the basic terminology necessary for further reading. The right (left) hand lay strand is a strand in which the cover wires are laid in a helix having a right (left) hand pitch. The right (left) hand lay rope is a rope in which the strands are laid in a helix having a right (left) hand pitch. The Lang's lay rope is a rope in which the wires in the strands are laid in the same direction that the strands in the rope are laid. The regular (ordinary) lay rope is a rope in which the wires in the strands and the strands in the rope are laid in opposite directions.

A steel wire as the basic structural member of strands and ropes can be laid in the strand or in the rope as:

- a core of the strand (the centreline of a core wire forms a straight line),
- a wire in a layer of the strand (the centreline of a wire in a layer forms a single helical curve),
- a strand core of the multi-strand rope (the centreline of a core wire forms a single helix), and
- a wire in a strand layer of the multi-strand rope (the centreline of a wire in a strand layer forms a double helical curve).

Consequently, at the mentioned cases the centreline of a wire forms from geometric aspect two types of helical curves (single and double helixes) depending on its location in the strand or in the rope. Direction is positive when the helix is right handed.

To generate the geometry of strands and ropes the centrelines of these helical curves must be expressed mathematically. The assembling of geometric transformations in the space is one of possibilities how to obtain the parametric equations of wire centreline (wire axis) [20].

3. Geometric transformations

Let us consider the Euclidean space over the real numbers field $\mathbf{E}^3(R)$ with a rectangular Cartesian coordinate system $(0; x, y, z)$. In this space every point M is uniquely determined by an ordered triplet of numbers $M[x_M, y_M, z_M]$, where x_M, y_M and z_M are the Cartesian coordinates of point M. By addition of ideal points to the basic Euclidean space, the extended Euclidean space $\mathbf{E}^3_\infty(R)$ in which the finite point M has homogeneous coordinates $M(x_M, y_M, z_M, 1)$ can be obtained.

The displacement of the point M to the point M' in the space can be formulated by the use of geometric transformation. Spatial transformation in the extended Euclidean space is analytically represented by square regular matrix of the 4th order with real elements. The matrices, which represent basic geometric transformations (translations and rotations of the coordinate system and the translation and rotations of the point) in the space, are specified in the Appendix A. Then, the homogeneous coordinates of the point $M'(x'_M, y'_M, z'_M, 1)$,to which the point $M(x_M, y_M, z_M, 1)$ is displayed by transformation $[T_i]$, are calculated as

$$\begin{Bmatrix} x'_M \\ y'_M \\ z'_M \\ 1 \end{Bmatrix} = [T_i]^T \begin{Bmatrix} x_M \\ y_M \\ z_M \\ 1 \end{Bmatrix} \tag{1}$$

where $[T_i]$ depends on the type of a translation or rotation (see the Appendix A). The assembly of several transformations is performed by the multiplication of the corresponding matrices $[T_i]$

$$\begin{Bmatrix} x'_M \\ y'_M \\ z'_M \\ 1 \end{Bmatrix} = \prod_{i=1}^{n} [T_i]^T \begin{Bmatrix} x_M \\ y_M \\ z_M \\ 1 \end{Bmatrix} \tag{2}$$