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Numerical characterization of the structural behaviour of the Basilica of Pilar in Zaragoza (Spain). Part 2: Constructive process effects

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Abstract

In the first part of this article, the finite element models developed for the Basilica of Pilar have been described, as well as the results obtained in linear theory and considering the material non-linearity. In the second part, a study concerning the constructive process and the geometrical non-linearity on the previous models is developed. Firstly we consider the procedure to include various construction stages in the finite element formulation of a structural problem. Then a geometrical non-linear analysis is applied to the two global models of the temple, the historical model of 1927 state and the actual state model, comparing the results obtained with the linear ones and explaining the problems encountered. Afterwards, a hypothetical constructive process is applied to the global models by means of the geometrical non-linearity, verifying and explaining the importance of this type of analyses in the obtained numerical results, especially in the Regina Martirum dome. Finally, applying the non-linear staged construction, the effects of an excavation ditch made in 1996 on the temple are considered.

Keywords: Historical buildings; Masonry; Structural models; Finite element analysis; Geometrical non-linearity; Non-linear staged construction

1. Introduction

The consideration of the effects of the constructive process is mandatory in the design of several types of structures like tunnels or excavation works, dam construction, long span bridges or high buildings [1,2]. Nevertheless, its inclusion in the analysis of historical structures is unusual, being difficult to find papers on the matter [3–5]. As it is verified later in the case of the Basilica of Pilar, the influence of considering the constructive process in the obtained results can be very important.

The historical constructive process has been incorporated in the study in two ways, using in both linear material models. In the first place, after modifying the initial 1927 model, with additional elements for the repairing works and new structures such as the river towers, we obtained the actual state model, which was again recalculated with the same loads [6]. The results of this approach are presented in part I of this article. In the second place we developed an evolutionary model considering several hypotheses about the historical construction process for both of them, the 1927 model and the actual state model, in order to verify if the numerical results are affected by the constructive process.

It is evident that in a massive structure, as the Basilica of Pilar, the effects of the geometric non-linearity, considering the deformed geometry in the structural analysis, may not be significant. However the simulation of the constructive process is based on a sequential analysis by stages or time steps, and the activation or deactivation of mesh elements in each analysis stage, starting from an initial mesh with all the elements involved in the analysis previously defined.

In each stage, the activated elements are incorporated into the analysis from that stage with a null initial

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stress–strain state, simulating new constructed parts originally deactivated. Once activated, their contributions to the global rigidity and loads are taken into account, along with the contributions of the rest of elements in an ‘‘alive’’ state. Deactivated or ‘‘killed’’ elements have a null elemental stiffness matrix, simulating the elimination of parts of construction as in excavation process, or structural elements still not constructed. Therefore, it is necessary to update the global stiffness matrix and load vector, in each stage, to consider the variation of contributions from the alive or killed elements. To accomplish this approach with the finite element program Cosmos/m [7] employed, we must include the geometrical non-linearity in order to force to the program to update these matrix, and each stage is considered as an independent analyses starting from the previous one through a restart. That allows the user to indicate the activated or deactivated elements between each stage. In other programs, like Sap2000 [8], it is possible to consider the non-linear staged construction effect with or without non-linear geometrical effects.

This type of analysis is clearly non-linear. The obtained results depend on the constructive sequence and the history of loads, and could be combined with other non-linearities coming from materials or geometrical, in a static or a dynamic context.

To avoid numerical instabilities, deactivated elements have usually a non-null value in the diagonal positions of their elementary stiffness matrix, but sufficiently reduced so that it does not affect the results of the active structure. In all the models developed in this paper we work with a residual stiffness value of 1×10^{-6} kN/m for deactivated elements. Similarly, the application of the loads must be made carefully, never applying loads on deactivated elements.

Usually finite elements programs do not allow us to modify the mesh in the different stages of an analysis. For that reason the structure is defined completely in the initial stage, indicating the elements that are deactivated. When an element is deactivated, its rigidity and its loads are not added to the analysis, and therefore their nodes do not undergo movements except in the case of having common nodes with active elements. It is important that the displacements of the active elements do not distort excessively the deactivated elements in contact to avoid erroneous results and numerical problems when these elements are activated. If the displacements undergone by the active elements are important, it is necessary to correct the position of the deactivate elements before their activation, for example, by means of rigid solid global movements placing these new elements in an appropriate position with respect to the already active ones.

2. Finite element formulation considering constructive process

Each constructive stage can be divided in a series of sequential steps with the purpose of improve the convergence. Assuming a geometrical non-linear analysis [9,10], the solution in each step is obtained in an iterative way by means of the Newton–Raphson method with force control.

The problem in step t and iteration i could be expressed as

$$ {}^{t}\mathbf{K}_{\mathrm{T}}^{i-1}\Delta\mathbf{a}^{i} = {}^{t}\mathbf{F} - {}^{t}\mathbf{R}^{i-1} \tag{1} $$

where ${}^{t}\mathbf{F}$ is the external load vector, $\Delta\mathbf{a}^{i}$ is the increment of displacements, and ${}^{t}\mathbf{K}_{\mathrm{T}}^{i-1}$, ${}^{t}\mathbf{R}^{i-1}$ are respectively the stiffness tangent matrix and the internal force vector, defined from the tensional state in iteration $i-1$.

2.1. Deactivation process

In order to simulate the deactivation of an element e from the mesh Ω, besides using a quasi null stiffness matrix in their assembly, an equivalent force to be applied in the active mesh $\mathbf{F}^{d}$ must be calculated considering two effects: first one due to the equivalent nodal forces produced by the loads of the element $\mathbf{F}_{p}^{d}$, and second one due to the stress state of the element in the previous stage to their deactivation $\mathbf{F}_{\sigma}^{d}$.

If n elements in contact with the node p are deactivated, the equivalent force associate to that node is

$$ \mathbf{F}_{p}^{d} = -\sum_{i=1}^{n}\mathbf{F}_{p}^{i} \tag{2} $$

Nodal forces of deactivation induced by the stress state are equal to the forces that the mesh exerts on the element:

$$ \mathbf{F}_{\sigma}^{d,e} = \int_{\Omega^{e}} (\mathbf{B}^{e})^{\mathrm{T}}\sigma^{e}\,\mathrm{d}\Omega \tag{3} $$

where $\mathbf{B}^{e}$ is the deformation matrix. In expression (1), if only the contributions of the active elements are considered in the assembly, the term $\mathbf{F}_{\sigma}^{d}$ appears directly in the vector of residual forces $\mathbf{F} - \mathbf{R}$, and it is only necessary to add $\mathbf{F}_{p}^{d}$ to the external load vector.

2.2. Activation process

When an element is activated, in addition to take in account its stiffness matrix and the equivalent nodal forces in the assembly process, in order to assure that its initial stress–strain state is null, all the element nodal movements must be null. Nevertheless, due to the discontinuous simulation by stages, the elements activated in intermediate stages could have non-null movements in the shared nodes with elements activated in previous stages. In this case a vector with the nodal movements associated to the element at the moment of its activation is defined. This vector of initial movements $\mathbf{a}_{0}$, is used in the calculation of stress and strains at the instant t by using the expression:

$$ {}^{t}\boldsymbol{\sigma}^{e} = \mathbf{D}^{e}\,{}^{t}\boldsymbol{\varepsilon}^{e} = \mathbf{D}^{e}\mathbf{B}^{e}({}^{t}\mathbf{a} - \mathbf{a}_{0}) \tag{4} $$

where $\boldsymbol{\varepsilon}^{e}$ is the element strain vector and $\mathbf{D}^{e}$ is the constitutive matrix.