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Improved FVM for two-layer shallow-water models: Application to the Strait of Gibraltar ☆

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Abstract

This paper deals with the numerical simulation of flows of stratified fluids through channels with irregular geometry. Channel cross-sections are supposed to be symmetric but not necessarily rectangular. The fluid is supposed to be composed of two shallow layers of immiscible fluids of constant densities, and the flow is assumed to be one-dimensional. Therefore, the equations to be solved are a coupled system composed of two Shallow Water models with source terms involving depth and breadth functions. Extensions of Roe's *Q*-scheme are proposed where a suitable treatment of the coupling and source terms is performed by adapting the techniques developed in [Vázquez-Cendón ME. Improved treatment of source terms in upwind schemes for the shallow water equations in channels with irregular geometry. J Comp Phys 1999;148:497–526; García-Navarro P, Vázquez-Cendón ME. On numerical treatment of the source terms in the shallow water equations. Comput Fluids 2000;29(8):17–45; Castro MJ, Macías J, Parés C. A *Q*-Scheme for a class of systems of coupled conservation laws with source term. Application to a two-layer 1-D shallow water system. Math Model Numer An 2001;35(1):107–27]. Finally we apply the numerical scheme to the simulation of the flow through the Strait of Gibraltar. Real bathymetric and coast-line data are considered to include in the model the main features of the abrupt geometry of this natural strait connecting the Atlantic Ocean and the Mediterranean Sea. A steady state solution is obtained from *lock-exchange* initial conditions. This solution is then used as initial condition to simulate the main semidiurnal and diurnal tidal waves in the Strait of Gibraltar through the imposition of suitable boundary conditions obtained from observed tidal data. Comparisons between numerical results and observed data and some tests on friction sensitivity are also presented.

Keywords: *Q*-schemes; Coupled conservation laws; Source terms; 1D shallow water equations; Two-layer exchange flows; Hyperbolic systems; Strait of Gibraltar; Internal tides; Fortnightly and monthly signal; Friction effects

1. Introduction

Two layer flows, or flows which can be idealized as such, occur naturally in estuary flows, ocean currents, and atmospheric flows. Two layer flows also occur as a result of man's interaction with natural flows by the addition of different density pollutants. This kind of flows are interesting from a theoretical and practical point of view. In this work, our interest focused on the computation of maximal and tidally induced two-layer exchange flows through the Strait of Gibraltar. In this narrows, connecting the Atlantic Ocean with the Mediterranean Sea, two layers of different waters can be distinguished: the colder and less saline Atlantic water flowing at surface and penetrating into the Mediterranean, and the deeper, denser Mediterranean water flowing into the Atlantic. The observation of this simplified picture shows that the simpler model to be used to simulate the flow in this region must unavoidably consider this two-layer structure.

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In the first part of this paper, the 1D two layer shallow water equations for channels with irregular geometry both in breadth and depth are presented. This general formulation extends the one-layer shallow water equations and takes into account all the sources and coupling terms due to the density difference between layers and also the terms due to the geometry. This kind of models has been widely used in oceanographical applications but mainly for channels with rectangular cross-sections. In this work a model taking into account more general cross-sections is considered.

The formulation presented is developed in the framework of hyperbolic conservation laws with source terms. A numerical method recently developed (see [9,8]) for this type of problems is considered. In particular, the numerical scheme is based on the use of a generalized Q-scheme to discretize the flux terms and an upwinding technique to discretize the source terms.

In previous works (see [9,8]) we assessed model performance by comparing model results against the approximate stationary analytical solutions provided [2,12] in channels with simplified geometries. In these works, *Armi and Farmer* characterize the concept of maximal solution for two-layer exchange flows through these simplified channels (a maximal solution is that allowing the maximal flux to be exchange through the channel fixed the ratio between the two layers fluxes). In the present contribution, a two-layer maximal exchange solution is computed for a realistic representation of the Strait of Gibraltar geometry and where an appropriate density ratio between the Mediterranean and Atlantic waters is chosen. The one-dimensional geometry considered for the Strait of Gibraltar is based on the consideration of a 1D symmetric channel with local sections *equivalent* to the actual sections of the Strait in a sense to be defined.

Then the main semidiurnal and diurnal tidal waves in the Strait of Gibraltar are simulated. This is done by taking the steady state solution of the previous experiment as initial condition and imposing the effect of tides through the boundary conditions at the open boundaries. The boundary conditions are constructed from tidal data obtained from [13]. The numerical results are validated against the observed data provided by the same authors and also compared with the synopsis of the essential elements of the time-dependent response of the flow in the Strait of Gibraltar made in [3] from observed data collected in April 1986.

Finally, some tests have been performed in order to assess model sensibility to friction and to determine how friction affects the amplitude of monthly and fortnightly signal appearing in model variable time series. The final aim is to compared these with the observed signals. This may be of interest for adjusting model friction coefficients. We conclude with some final remarks.

2. Governing equations

In [10,8] the general equations governing the one-dimensional flow of two shallow layers of immiscible fluids along a straight channel with symmetric cross-sections of arbitrary shape were deduced. We address the reader to the former reference for further details. The system of equations is written under the form of two coupled systems of conservation laws with source terms in the sense introduced in [9].

Let us introduce some notation. In general, index 1 makes reference to the upper layer and index 2 to the lower one. The coordinate x refers to the axis of the channel; y is the horizontal coordinate normal to the axis; z, the vertical coordinate; and t, the time; g is the gravity; ρ_i is the density of the ith layer ($\rho_1 < \rho_2$), and $r = \frac{\rho_1}{\rho_2}$, their ratio. Variables $b(x)$ and $\sigma(x,z)$ are, respectively, bottom and breadth functions, i.e., channel bottom is defined by the surface of equation $z = b(x)$ and channel walls by the equations: $y = \pm\frac{1}{2}\sigma(x,z)$. The variables $A_i(x,t)$ and $h_i(x,t)$ represent the wetted cross-section and the thickness of the ith layer at the section of coordinate x at time t (see Fig. 1), respectively. Therefore A_i and h_i, $i = 1,2$ are related through the equations:

$$A_1 = \int_{b(x)+h_2}^{b(x)+h_1+h_2} \sigma(x,z)\,\mathrm{d}z, \tag{1}$$

$$A_2 = \int_{b(x)}^{b(x)+h_2} \sigma(x,z)\,\mathrm{d}z. \tag{2}$$

Finally, $v_i(x,t)$ and $Q_i(x,t) = v_i(x,t)A_i(x,t)$ represent the velocity and the discharge of the ith layer.

The general equations governing the one-dimensional flow of two shallow layers of immiscible fluids along a straight channel with symmetric cross-sections of arbitrary shape are written as follows (see [8] for details):

$$\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x}(\boldsymbol{\sigma},\mathbf{W}) = \mathbf{B}(\boldsymbol{\sigma},\mathbf{W})\frac{\partial \mathbf{W}}{\partial x} + \mathbf{V}(\boldsymbol{\sigma},\mathbf{W}) + \mathbf{S}(x,\boldsymbol{\sigma},\mathbf{W}), \tag{3}$$

where

$$\mathbf{W}(x,t) = [A_1(x,t), Q_1(x,t), A_2(x,t), Q_2(x,t)]^{\mathrm{T}}, \tag{4}$$

$$\boldsymbol{\sigma}(x,t) = [\sigma_1(x,t), \sigma_2(x,t), \sigma_3(x,t)]^{\mathrm{T}}, \tag{5}$$

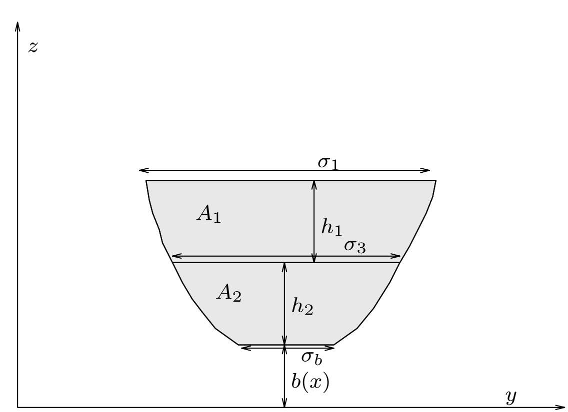

Fig. 1. Notations for channel cross-sections.