Available online at www.sciencedirect.com
ScienceDirect
Advances in Engineering Software 38 (2007) 410–422
ADVANCES IN ENGINEERING SOFTWARE
www.elsevier.com/locate/advengsoft

GLANUSIT: A software toolbox for the numerical simulation of large ice masses evolution ☆

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Available online 25 October 2006

Abstract

The here presented GLAciology NUmerical SImulation Toolbox (GLANUSIT) is a software application which provides a user friendly environment for the numerical simulation of large ice masses evolution. The graphical user interface has been developed in MATLAB while the core of GLANUSIT contains the original FORTRAN codes, which develop the specific numerical methods for the solution of the complex shallow ice model. This highly nonlinear model governs the coupled thermodynamical and hydrodynamical processes. The global algorithm mainly consists on a fixed point iteration between the different subproblems. The numerical solution of each subproblem requires specific techniques, which are not common in present software packages, as for example the part of moving boundaries solvers included in the code. Finally, a practical case study with real data is presented.

MSC: 68N15; 68N19; 68U20; 35K60; 65N30

Keywords: Object oriented methodology; Software application; User interface; Shallow ice model; Nonlinear problems; Numerical simulation; Finite elements

1. Introduction

As in other complex processes, recent advances in computer technologies make it possible the numerical simulation of the highly nonlinear mathematical models which govern the thermodynamics and mechanics of large ice masses. Some difficulties of these models arise from the fact that the involved mathematical equations are posed over unknown domains. Thus, not only the values of the physical magnitudes but also the domains and interfaces (moving boundaries) between them have to be identified. Moreover, the global mathematical model must describe the occurrence of several coupled processes. Thus, nonlinear thermodynamical, hydrodynamical and thermomechanical models are closely related in a feedback procedure and require specific well-suited numerical techniques.

The formulation of thermomechanical models which explain the behaviour of large ice masses are introduced in [1], for example. By applying the shallow ice approximation to the original conservation laws of continuum mechanics for ice, a simplified model for the temperature and an isothermal model for the profile are posed in [2]. From this departure paper, in recent works [3–6] these models have been extended to incorporate additional moving boundaries and nonlinear features. All these works include appropriate and specific numerical simulation algorithms for each subproblem. In the present paper we address the numerical solution of the complex coupled problem and we also add the treatment of new nonlinear terms entering in the Signorini boundary condition at the

☆ This work was partially supported by Research Projects MTM2004-5796-C02 and Xunta de Galicia SXID-2002-PR405A and PGIDIT02PXIC10503PN.
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0965-9978/$ - see front matter
doi:10.1016/j.advengsoft.2006.09.009

ice sheet base. Both the highly specific techniques and the complexity of the coupling between submodels explain the fact that (to our knowledge) there is no software toolbox for the numerical simulation of the coupled process. A possible alternative, which consists of the solution of the even more complex departure continuum mechanics models [7], might be offered by certain commercial codes (FLUENT, FEMLAB, etc.). Nevertheless, in such approach, the equations become more complicated and the presence of several moving boundaries is neither available in the models nor in the codes. GLANUSIT software application is motivated by these necessities.

As most finite element software libraries, the numerical methods developed in GLANUSIT core are written in FORTRAN programming language. The existence of periodically improved FORTRAN compilers for personal computers motivates the choice of this language for the GLANUSIT core implementation. On the other hand, to present GLANUSIT software application in a user-friendly environment, MATLAB multifunctional system has been selected. Note that there exist several MATLAB toolboxes (PDE Toolbox, FEMLAB) to solve (systems of) partial differential equations with some finite element methods [8], which do not include those handled by GLANUSIT. The appearance of their graphical user interfaces has been taken as a reference. The combination of numerical computing in FORTRAN codes with the interface in MATLAB environment can be additionally justified by several reasons. First, the use of MATLAB in finite element computations becomes very slow for fine enough meshes to obtain the required accurate results. Secondly, the use of FORTRAN90 for the interface programming part should be a possible alternative, more tedious than the easier to handle MATLAB commands. Thirdly, MATLAB allows to execute FORTRAN codes as internal functions.

The plan of the paper is as follows. In Section 2, the main points of the shallow ice global model are described, the mathematical equations are posed and the numerical techniques for simulation are briefly indicated. In Section 3, the object oriented methodology for the design and implementation of GLANUSIT is analyzed. A case study is presented in Section 4 and some conclusions are summarized in Section 5.

2. The shallow ice model and its numerical simulation techniques

The glaciology mathematical model used by GLANUSIT can be framed in the well-known *shallow ice approximation* of the initial continuum mechanics based models (mainly posed in terms of mass, momentum and energy conservation laws) [1]. The idea of this approach is to scale appropriately (*shallow ice scaling*) the coordinates and unknowns so that some terms could be neglected in the dimensionless equations, by taking into account that length and width are far larger than depth in a typical ice sheet geometry. For the Antarctic ice sheet, typical length and width values are $d_1 = 3\times10^6$ m while depth is about $d_2 = 3\times10^3$ m, so that the aspect ratio $\epsilon = d_2/d_1 = 10^{-3}$ is small enough to neglect $O(\epsilon^2)$ terms in the dimensionless conservation laws in order to pose simpler asymptotic models. Moreover, as an almost bidimensional flux occurs, we can restrict ourselves to a longitudinal section of the initial 3-d ice sheet geometry. For a justified statement of the shallow ice models here proposed for the profile, velocity and temperature, we address the reader to the works [2,3,6,10]. The different partial models have been previously solved separately by the authors in previous works: a simpler version of the thermal problem in [3], two approaches of the profile problem in [4,9] the coupling between both for a prescribed velocity field in [5,6], for example. In this paper, a particular novelty is the fully implicit treatment of the nonlinear Signorini boundary condition when including additional nonlinear sliding terms. For the numerical solution of this original coupled complex model, in a first step a fixed point iteration uncouples the different submodels. Secondly, appropriate specific techniques are applied to solve sequentially the different difficulties associated to each submodel.

2.1. The profile model simulation

Once shallow ice approximation has been performed, in this paragraph we pose the dimensionless problem that provides the upper profile of an ice sheet section. For this, in order to pose the dimensionless coupled problem, a large enough fixed rectangular domain Ω_G is considered. It includes not only a longitudinal section of the ice sheet but also the part of the atmosphere above the ice mass. Thus, for a large enough value z_{max}, the domain is defined by

$$\Omega_G = \{(x,z)/-1 \leqslant x \leqslant 1,\ 0 \leqslant z \leqslant z_{max}\} \tag{1}$$

and the ice mass domain is given, in terms of the unknown upper profile η, by

$$\Omega_I(t) = \{(x,z)/S_-(t) \leqslant x \leqslant S_+(t),\ 0 \leqslant z \leqslant \eta(t,x)\}, \tag{2}$$

so that, for any time t, the inclusion $\Omega_I(t) \subset \Omega_G$ holds. Therefore, we can write

$$\Omega_G = \Omega_I(t) \cup \Omega_A(t), \tag{3}$$

where $\Omega_A(t)$ denotes the atmospheric domain. Notice that, since the ice layer longitudinal extent is not fixed, the interval $(S_-(t), S_+(t)) \subset (-1,1)$ is an additional unknown to the profile function η. The presence of this unknown boundaries, $S_-(t)$ and $S_+(t)$, leads to the moving boundary feature. Thus, the moving boundary profile problem is posed over the fixed domain $\Omega = (-1,1)$. For this, let $t_{max} > 0$ be a large enough time instant and let $a : (0, t_{max}) \times \Omega \to \mathbb{R}$ and $\eta_0 : \Omega \to \mathbb{R}$ be the given accumulation-ablation rate and initial profile, respectively. Then, for $t \in [0, t_{max}]$, the formulation can be stated as follows [4]:

Find $\Gamma_0(t) = (S_-(t), S_+(t)) \subset \Omega$ and $\eta : (0, t_{max}) \times \Omega \to \mathbb{R}$ such that: