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A remedy to gradient type constraint dilemma encountered in RKPM

Hossein M. Shodja *, Alireza Hashemian

Center of Excellence in Structures and Earthquake Engineering, Department of Civil Engineering, Sharif University of Technology, Azadi Avenue, P.O. Box 11365-9311, Tehran, Iran

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Abstract

A major disadvantage of conventional meshless methods as compared to finite element method (FEM) is their weak performance in dealing with constraints. To overcome this difficulty, the penalty and Lagrange multiplier methods have been proposed in the literature. In the penalty method, constraints cannot be enforced exactly. On the other hand, the method of Lagrange multiplier leads to an ill-conditioned matrix which is not positive definite. The aim of this paper is to boost the effectiveness of the conventional reproducing kernel particle method (RKPM) in handling those types of constraints which specify the field variable and its gradient(s) conveniently. Insertion of the gradient term(s), along with generalization of the corrected collocation method, provides a breakthrough remedy in dealing with such controversial constraints. This methodology which is based on these concepts is referred to as gradient RKPM (GRKPM). Since one can easily relate to such types of constraints in the context of beam-columns and plates, some pertinent boundary value problems are analyzed. It is seen that GRKPM, not only enforces constraints and boundary conditions conveniently, but also leads to enhanced accuracy and substantial improvement of the convergence rate.

Keywords: Meshless; RKPM; Gradient; Constraint; Beam-column

1. Introduction

In many problems of computational physics and engineering, one needs to incorporate the gradients of the field variables accurately. This task, in the context of reproducing kernel particle method (RKPM) [1], requires the increase in number of particles together with employment of higher order correction functions. An important area that RKPM needs improvement relates to the essential boundary conditions (EBCs) involving the derivatives of the field quantities. Unlike finite element method (FEM), the shape function associated with RKPM does not necessarily satisfy the Kronecker delta property. Hence, enforcement of the EBCs via conventional RKPM becomes inconvenient. The present paper, based on introduction of the derivatives of the function into the reproducing equation, develops new formulations of the RKPM for one- and two-dimensional (1D and 2D) problems, which is referred to as gradient RKPM (GRKPM). It will be shown that, the proposed approach can conveniently incorporate the EBCs involving the derivatives of the function, and produce more accurate results than the conventional one.

Suppose that for a one-dimensional problem of the interest, the EBCs involve both the function and its first derivative

$$u(x) = g_0(x), \quad x \in \Gamma_0, \tag{1a}$$
$$u'(x) = g_1(x), \quad x \in \Gamma_1, \tag{1b}$$

where Γ_0 is that part of the boundary on which $u(x)$ is prescribed, whereas Γ_1 refers to the boundary on which $u'(x)$ has been specified. In the context of meshfree methods, i.e., RKPM, element free Galerkin method (EFGM) [2], and generalized moving least squares [3], assuming

$$\psi_{ij}(x) \equiv \psi_j(x_i) \neq \delta_{ij}, \tag{2}$$

* Corresponding author. Tel.: +98 21 6600 5818; fax: +98 21 6601 4828.
E-mail address: shodja@sharif.edu (H.M. Shodja).

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where $\psi_j(x_i)$ is the shape function associated with the jth particle, which is evaluated at the ith particle, and δ_{ij} is the Kronecker delta. In the above-mentioned meshfree methods, the function $u^{\mathrm{R}}(x)$, which is the reproduced function of $u(x)$, is related to the values of degrees of freedom (DOF) $\mathbf{d}$ through $u^{\mathrm{R}}(x) = \boldsymbol{\psi}(x)\mathbf{d}$. For this reason in view of (2) at a given point x_i, the value of u and the value of degree of freedom are not equal, i.e., $u(x_i) \neq d_i$. It is due to this undesirable property which makes it cumbersome to relate the value of $u(x)$ at an essential boundary node to the value of the pertinent degree of freedom. Similar difficulty holds for the derivative of the function, $u'(x)$.

The above-mentioned shortcomings of RKPM, have posed a challenge and received the attentions of many distinguished investigators to this issue, subsequently in the literature, there are many publications devoted to the subject. For example Gosz and Liu [4] have employed a type of correction function which takes on the value of zero on the boundaries, and Günther and Liu [5] have used a scheme based on d'Alembert's principle. In the context of EFGM, the method of Lagrange multiplier was first proposed by Belytschko et al. [2], and modified by Lu et al. [6]. This method has been extensively used to enforce EBCs including the field quantity and its first derivative(s), e.g. in plate problems [7,8]. The penalty method is another alternative for this purpose [3], which was first proposed by Belytschko et al. [9] and detailed by Zhu and Atluri [10]. Chen et al. [11] presented the transformation technique which is very efficient. However, it is not applicable to the derivative type of essential boundary conditions. Krongauz and Belytschko [12] have employed EFGM to the interior domain, and applied FEM to the strips in the neighborhood of the boundaries. Thorough discussions on the advantages and shortcomings of these methods are given by Li and Liu [13].

2. Development of 1D GRKPM

2.1. Reproducing equation

Let the reproduced function be expressed in terms of the function and its first derivative

$$u^{\mathrm{R}}(x) = \int_{\Omega} \bar{\phi}_a^0(x; x-y) u(y)\,\mathrm{d}y + \int_{\Omega} \bar{\phi}_a^1(x; x-y) u'(y)\,\mathrm{d}y, \tag{3}$$

where $u(y)$ is a field quantity whose gradient is $u'(y)$, $\bar{\phi}_a^0(x; x-y)$ and $\bar{\phi}_a^1(x; x-y)$ are the modified kernel functions associated with the function and its gradient, respectively.

$$\bar{\phi}_a^0(x; x-y) = C^0(x; x-y)\phi_a(x-y), \tag{4a}$$

$$\bar{\phi}_a^1(x; x-y) = C^1(x; x-y)\phi_a(x-y). \tag{4b}$$

In relation (4) a is a dilation parameter, $\phi_a(x-y) = \frac{1}{a}\phi\left(\frac{x-y}{a}\right)$ is a kernel function, $C^0(x; x-y)$ and $C^1(x; x-y)$ are the correction functions defined by

$$C^0(x; x-y) = \sum_{s=0}^{n} \xi_s(x)(x-y)^s, \tag{5a}$$

$$C^1(x; x-y) = \sum_{s=0}^{n} \eta_s(x)(x-y)^s, \tag{5b}$$

where ξ_s's and η_s's are the unknown coefficients, which are determined from the completeness conditions. Eqs. (4) and (5) yield

$$\bar{\phi}_a^0(x; x-y) = \sum_{s=0}^{n} \xi_s(x)(x-y)^s \phi_a(x-y), \tag{6a}$$

$$\bar{\phi}_a^1(x; x-y) = \sum_{s=0}^{n} \eta_s(x)(x-y)^s \phi_a(x-y). \tag{6b}$$

2.2. Completeness

2.2.1. Conditions on the function

Consider the first $2n+2$ terms in the Taylor series expansion of u about point x

$$u(y) \cong u(x) + \sum_{\alpha=1}^{2n+1} \frac{(-1)^\alpha (x-y)^\alpha}{\alpha!} u^{(\alpha)}(x), \tag{7}$$

where

$$u^{(\alpha)} = \frac{\mathrm{d}^\alpha u}{\mathrm{d}x^\alpha}. \tag{8}$$

Differentiating (7), gives

$$u'(y) \cong -\sum_{\alpha=1}^{2n+1} \frac{(-1)^\alpha \alpha (x-y)^{\alpha-1}}{\alpha!} u^{(\alpha)}(x). \tag{9}$$

Upon substitution of Eqs. (7) and (9) into (3), one obtains

$$u^{\mathrm{R}}(x) \cong u^{\mathrm{R0}}(x) + u^{\mathrm{R1}}(x), \tag{10}$$

where

$$u^{\mathrm{R0}}(x) = \sum_{\alpha=0}^{2n+1} \left[\frac{(-1)^\alpha}{\alpha!} u^{(\alpha)}(x) \int_{\Omega} (x-y)^\alpha \bar{\phi}_a^0(x; x-y)\,\mathrm{d}y \right], \tag{11a}$$

$$u^{\mathrm{R1}}(x) = \sum_{\alpha=1}^{2n+1} \left[\frac{(-1)^\alpha}{\alpha!} u^{(\alpha)}(x) \int_{\Omega} -\alpha (x-y)^{\alpha-1} \bar{\phi}_a^1(x; x-y)\,\mathrm{d}y \right]. \tag{11b}$$

In view of relations (6) and (11), one may write

$$u^{\mathrm{R0}}(x) = \sum_{\alpha=0}^{2n+1} \left[\frac{(-1)^\alpha}{\alpha!} u^{(\alpha)}(x) \sum_{s=0}^{n} \xi_s(x) m_{\alpha+s}(x) \right], \tag{12a}$$

$$u^{\mathrm{R1}}(x) = \sum_{\alpha=0}^{2n+1} \left[\frac{(-1)^\alpha}{\alpha!} u^{(\alpha)}(x) \sum_{s=0}^{n} -\alpha \eta_s(x) m_{\alpha+s-1}(x) \right], \tag{12b}$$

where $m_k(x)$ is the kth moment of the kernel function

$$m_k(x) = \int_{\Omega} (x-y)^k \phi_a(x-y)\,\mathrm{d}y. \tag{13}$$