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Generation of hierarchical multi-resolution medial axis for CAD models

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ABSTRACT

Medial axis (MA) is used as an effective description for objects in many engineering fields. A difficulty for the current methods for the generation of MA of CAD models is the balance between the efficiency and the quality. In this study, an approach to iteratively generating hierarchical multi-resolution MA is proposed. In each iteration, only a small part of MA that affects MA quality is refined, by which the time cost and the space cost are reduced greatly. First, the model is voxelized and its initial MA is generated by distance dilation method. Meanwhile, the MA quality is computed and evaluated. Second, if the MA quality does not satisfy the requirement, upgrade the MA level and re-compute the local MA in the affected region until the MA quality does. Finally, by combining the local MA in the affected region with the reused MA in other regions, hierarchical multi-resolution MA is obtained. Several examples are given to demonstrate the outperformance of the proposed method in terms of time and space.

1. Introduction

Medial axis (MA) is first proposed by Blum [1] as an effective description for objects. The MA of an object is the set of all points having more than one closest point on the object's boundary [2]. It has been widely used in many engineering fields, such as finite element analysis, shape analysis, robot route planning, solid modeling, and mesh generation [3–5]. For a complicated CAD model, as the computational complexity is too high, it is difficult to compute its accurate MA. Therefore, researchers usually use approximation methods to obtain the MA of a CAD model. For example, the surface-based method [6] computes approximate MA represented by points. The MA usually consists of some MA segments, each of which is generated by one or two boundary faces. Here, an MA segment here may refer to a curve edge/face/point based on the context. Moreover, the connection between MA segments can be obtained directly during the generation process by checking common MA voxels of different MA segments. However, the resultant MA of the surface-based method is difficult to be transformed to MA segments.

Another work is thinning method based on distance transformation [7]. During the thinning process of a CAD model, the input model is usually first discretized into some voxels under a certain resolution, and then the MA represented by voxels is computed. However, it is not easy to satisfy both the requirement of the MA quality and the computational efficiency at the same time in the computation of the MA of a CAD model. If the model resolution is low, the MA quality will be low since the details of the MA cannot be properly represented by large voxels although the computational efficiency is high. On the contrary, the time cost and space cost will be intolerable although the MA quality is high if the model resolution is high. How to balance the quality and the efficiency is a difficult problem during the MA generation. To control the resolution, Gao et al. [8] proposed a method of multi-resolution MA generation. In their method, the concept "dilation unit" is used to combine the small boundary faces with large boundary faces to ignore their impact on the MA. Their method can efficiently suppress noises and thus improve the MA robustness. However, since the voxel resolution is still constant in this method, the MA segments generated by non-noise small boundary faces that have to be concerned still cannot be well computed efficiently.

Based on above analysis, a method is proposed to compute the hierarchical MA of a CAD model with the help of multi-resolution voxels. The main idea is that based on the initial MA, local MA is refined iteratively to improve the MA quality until the requirement is satisfied. In the hierarchical MA, high-resolution voxels are used for the small boundary faces and MA segments generated by them to embody the details while normal-resolution voxels are for other boundary faces and MA segments generated by them. Therefore,

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high-quality MA can be generated at both low time cost and low space cost. The novelty of this study will be reflected in the following aspects. (1) A method to evaluate the MA quality is proposed based on boundary faces and MA segments. (2) In each model level, to refine the model based on the voxelized model efficiently, a method is proposed to determine the details of the model, called the re-voxelization region and conduct multi-resolution voxelization in the re-voxelization region by high-level voxels. (3) The affected region is proposed only in which the local MA needs to be refined to improve the MA quality. By combining the new local MA to the reused MA, the hierarchical MA is obtained. (4) The adaptive distance dilation is proposed to refine MA voxels by using boundary voxels in re-voxelization region. Based on the adaptive distance dilation, an adaptive double queues distance dilation based algorithm is proposed, which can be used to compute voxel distances for generating MA voxels.

As the re-voxelization region and the affected region are both much smaller than the entire model, not only the high-quality MA can be achieved, but also low time complexity and space complexity is ensured.

This paper is organized as follows. Related work is introduced in Section 2 while the basic concept and method overview are introduced in Section 3. In Sections 4, 5 and 6, the multi-resolution voxelization, computation of multi-resolution voxel distances and multi-resolution MA generation method are detailed, respectively. The complexity, robustness and accuracy are analyzed in Section 7. Some examples are illustrated in Section 8 while conclusions are given in Section 9.

2. Related work

The traditional methods of MA generation can be divided into three categories: thinning method, tracing method and Voronoi-graph based method. Here only the thinning method which is related to this paper is introduced. For more detail information about MA, readers can refer to Ref. [9].

In thinning method, a model is usually first voxelized to achieve its MA. The error of resultant MA is decided by the resolution of voxelization. Lam et al. [10] introduced thinning method systematically. Nackman [5] proposed a method which replaces the smooth boundary by a polygon/polyhedron, and thus MA of polygons can be used as MA of the input model. Based on the diffusion of the combined waves, Scott et al. [11] proposed a method to compute the symmetric axis of a method as a superset of the MA. This method is very effective for 0-1 images, but the computational error is considerably greater for color images. Siddiqi et al. [12] proposed a vector field-based thinning method. In the approach by Vleugels et al. [13], voxelization was recursively performed on a space that contained MA voxels until the necessary resolution was reached. To preserve the topology of the model, Borgefors et al. [14] proposed a thinning method for 3D skeletonization that can be used to compute both the surface skeleton and the curve skeleton. In their method, the Euclidean distance can be represented by the local distances, and an optimal method is given to compute the minimum distance between two points [15].

To reduce the computational complexity, many researchers have proposed various strategies. Sherbrooke et al. [16] built the linearly approximate segments of the curve in the direction of the tracing. Their method can be used for conveniently computing the MA of polyhedrons. Using a piecewise circular boundary conversion, Aichholzer et al. [17] handled models with free-form shapes to efficiently generate the MA. Moreover, with their method, convergence is guaranteed. To speed up the Voronoi graph-based MA generation method, Meijster et al. separated the grid points of the image into independent rows and columns; thus, it is well suited for parallelization on a shared-memory machine [18]. Hirata devised an efficient algorithm for each row to find the lower envelope of the minima of the set of distance functions [19].

Some researchers also considered the generation of multi-resolution MA. Miklos et al. [20] proposed a method which generated the MA in different abstract levels. Reniers et al. [21] robustly computed line skeleton and curve skeleton for 3D models under multiple levels. However, those methods [20,21] cannot represent MA for details of the model efficiently. Gao et al. [8] also proposed a method to generate multi-resolution MA based on distance-dilation method in which the MA voxel size is constant. When MA voxel size is small to represent details of the model, the computational cost of their method will be too high to be implemented.

Another typical work is the parallel computation of MA generation by multiple processors. A parallel banding algorithm based on the GPU was proposed in Ref. [22]. In this method, exact Euclidean distance transform (EDT) was computed efficiently for 2D images. However, the degree of parallelization is low when the texture sizes are high. Moreover, many researchers also proposed similar GPU based parallel methods [6,23–26]. Recently, a parallel method based on multi-CPU for MA generation is proposed [27]. However, those methods cannot generate high-quality MA that maps to MA segments.

3. Basic concept and method overview

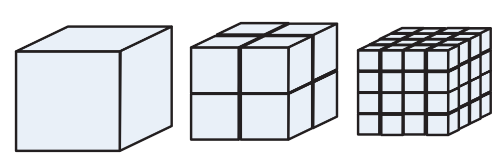
3.1. Basic concept

To facilitate the reader's understanding, some related basic concepts are introduced here first here.

Definition 1. The *distance of voxel A* means the distance between voxel A and its nearest boundary voxel. The nearest boundary voxel is called touch voxel of voxel A [7]. Here the distance between two voxels refers to the Euclidean distance between their centers. Particularly, the distance of a boundary voxel is 0 and its touch voxel is itself.

Definition 2. Hierarchical *multi-resolution voxels* are hierarchical voxels which have different voxel sizes. Initially, a voxel of level 0 (called L_0 voxel for short) represent a voxel in basic resolution, which is also called a basic voxel with edge size of E_0. Similarly, the edge size of an L_i voxel is called E_i. Here, $E_i = E_0/2^i$. A basic voxel can be virtualized and be divided into 8^i $*L_i$ child voxels, which means that the *refining level* of the basic voxel is i (called R_i basic voxel for short). As shown in Fig. 1a, an R_0 basic voxel A is shown. When the refining level of basic voxel A is upgraded to 1, it is divided into 8 L_1 child voxels as shown in Fig. 1b. Then, when the refining level of basic voxel A is upgraded to 2, it is divided into 64 L_2 child voxels as shown in Fig. 1c.

Definition 3. *The MA level* represents the hierarchy of MA of different qualities. The MA level of initial MA is 0, which means that the model only contains L_0 voxels. When the MA level is upgraded to i gradually, the model will contain L_i voxels. When the MA level is



(a) An R_0 basic voxel (b) An R_1 basic voxel (c) An R_2 basic voxel

Fig. 1. The hierarchical multi-resolution voxels.