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A response surface approach for structural reliability analysis using evidence theory

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ABSTRACT

Evidence theory employs a much more general and flexible framework to quantify the epistemic uncertainty, and thereby it is adopted to conduct reliability analysis for engineering structures recently. However, the large computational cost caused by its discrete property significantly influences the practicability of evidence theory. This paper proposes an efficient response surface (RS) method to evaluate the reliability for structures using evidence theory, and hence improves its applicability in engineering problems. A new design of experiments technique is developed, whose key issue is the search of the important control points. These points are the intersections of the limit-state surface and the uncertainty domain, thus they have a significant contribution to the accuracy of the subsequent established RS. Based on them, a high precise radial basis functions RS to the actual limit-state surface is established. With the RS, the reliability interval can be efficiently computed for the structure. Four numerical examples are investigated to demonstrate the effectiveness of the proposed method.

1. Introduction

Uncertainties related to the material property, bounding condition, load, etc. widely exist in practical engineering problems. With intensive requirements of high product quality and reliability, understanding, identifying, controlling and managing various uncertainties have become imperative. Uncertainty refers to the difference between the present state of knowledge and the complete knowledge. Based on this view, uncertainty can be described as two distinct types – aleatory (random) and epistemic (subjective) uncertainty [1]. *Aleatory uncertainty* is irreducible and describes the inherent variability of a physical system, which can be modeled as random variables or processes using probability theory. Many probability-based reliability analysis techniques have been well established and successfully applied to varieties of industrial fields [2–5]. However, when data are scarce, the probability theory becomes not so useful because the key probability distributions cannot be obtained. In this case, the epistemic uncertainty will be involved. *Epistemic uncertainty* is defined as the lack of knowledge or information in some phases or activities of the modeling process. Therefore, it can be reduced with the collection of more information or an increase of knowledge. Some representative theories, including convex models [6–11], possibility theory [12–14], fuzzy sets [15] and evidence theory [16–21], can be used to deal with the epistemic uncertainty.

Among the above theories for epistemic uncertainty, evidence theory employs a much more flexible framework with respect to the body of evidence and its measures [22]. Under some special situations, it can provide equivalent descriptions to the probability theory, convex models, possibility theory and fuzzy sets, respectively. Hence, in recent years evidence theory has been introduced to conduct reliability analysis and design for engineering structures and mechanical systems. Oberkampf and Helton [22] compared the similarities and differences between evidence theory and probability theory in reliability analysis through a simple algebraic function. Helton et al. [23] explored several approaches (probability model, evidence theory, possibility theory and interval analysis) in the representation of the uncertainty in model prediction and thereby gave a unified framework. Soundappan et al. [24] compared evidence theory with Bayesian theory in aspects of uncertainty modeling and decision making under epistemic uncertainty. Du [25] formulated a new reliability analysis model to handle the epistemic and aleatory mixed uncertainty. Tonon et al. [26] employed evidence theory to quantify the parameter uncertainty in rock engineering and whereby carried out a reliability-based design of tunnels. Through creating a multi-point approximation at a certain point on the limit-state surface, Bae et al. [27,28] proposed an efficient reliability analysis method for structures with epistemic uncertainty. Jiang et al. [29] proposed a structural reliability method using evidence theory by introducing a non-probabilistic reliability index approach. Agarwal et al. [30] proposed an evidence-theory-based multidisciplinary design optimization (EBDO) algorithm through a sequential approximate strategy. Alyanak

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et al. [31] adopted a gradient projection technique to conduct a reliability-based design optimization (RBDO) for structures with epistemic uncertainty. Helton et al. [32] developed a sampling-based approach for sensitivity analysis of the uncertainty propagation problems using evidence theory. Mourelatos and Zhou [33] proposed a RBDO method based on evidence theory. Guo et al. [34] developed a RBDO method by combining evidence theory and interval analysis. Bai et al. [35] compared three metamodeling techniques for evidence-theory-based reliability analysis through six numerical examples.

Despite the above achievements, presently evidence theory has been barely applied to conduct reliability analysis for complex engineering problems. One main reason is the high computational cost caused by the discontinuous nature of uncertainty quantification for the evidence variable [26]. Unlike the probability density function (PDF) in probability model, the uncertainty modeled by evidence theory is propagated through a discrete basic probability assignment (BPA), which cannot be expressed by any explicit function but generally described by a series of discontinuous subsets. This will in general lead to a combination explosion difficulty for a multidimensional problem. By using the response surface of the actual limit-state function, the high computational cost of evidence-theory-based reliability analysis can be significantly reduced. Some numerical methods [27,28] have been developed to reduce the computational cost by introducing the response surface technique, however, it seems not always an easy job to construct a sufficiently accurate response surface for a practical engineering problem using the existing methods. Therefore, to improve the applicability of evidence theory in practical applications, it seems necessary to develop some more robust and efficient reliability analysis methods.

In this paper, a new response surface method is proposed to significantly improve the computational efficiency of evidence-theory-based reliability analysis, in which the analysis precision can be well guaranteed through a design of experiments technique. The remainder of this paper is organized as follows. The conventional reliability analysis using evidence theory is introduced in Section 2. An efficient algorithm is formulated to assess the reliability in Section 3. Four numerical examples are investigated in Section 4. Finally some conclusions are summarized in Section 5.

2. Conventional reliability analysis using evidence theory

In this section, a simple problem is used to show the conventional reliability analysis using evidence theory, in which some fundamentals of evidence theory will also be introduced.

Consider the following two-dimensional limit-state function:

$$g(\mathbf{X}) = g_0 \tag{1}$$

where $\mathbf{X} = (X_1, X_2)$ is the vector of two independent uncertain input parameters; g_0 denotes an allowable value of the structural responses. For this problem, the safety region G is defined as:

$$G = \{g : g(\mathbf{X}) \geqslant g_0\} \tag{2}$$

In this paper, the uncertain parameters $\mathbf{X}$ will be described using evidence variables, and the reliability interval that $\mathbf{X}$ falls into the safety region G can be computed by two main steps.

2.1. Construction of joint BPA

Evidence theory starts by defining a *frame of discernment* (FD) that is a set of mutually exclusive elementary subsets, which is similar to the sample space in probability theory. Here, the symbol X used to denote a parameter also represents its FD. All the possible subsets of X will form a power set $\Omega(X)$.

After defining the FD, a degree of belief is assigned to each subset based on the statistical data or the expert experience. It is called the *basic probability assignment* (BPA). The BPA is assigned through a mapping function $m:\Omega(X) \to [0, 1]$ which satisfies the following three axioms:

$$\begin{aligned} &Axiom\ 1:\ m(A) \geqslant 0 \text{ for any } A \in \Omega(X) \\ &Axiom\ 2:\ m(\emptyset) = 0 \\ &Axiom\ 3:\ \sum_{A\in\Omega(X)} m(A) = 1 \end{aligned}$$

where $m(A)$ characterizes the amount of "likelihood" that is assigned to the subset A. In this paper, we assume that the subsets A are all closed intervals instead of some other forms of sets. Each set $A \in \Omega(X)$ satisfying $m(A) > 0$ is called *focal element*. Sometimes the information available for a parameter may come from multiple sources, for example several experts provide opinions for one event, then they should be aggregated by rules of combination [36].

Similar to joint probability in probability theory, the *joint BPA* is required in evidence theory when multiple uncertain variables are involved. Due to the independence among the parameters, the joint BPA m_X can be obtained for a two-dimensional problem:

$$m_X(C) = \begin{cases} m_{X_1}(A) \cdot m_{X_2}(B) & \text{when } C \in A \times B \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where $A \in \Omega(X_1)$, $B \in \Omega(X_2)$, and C is the focal element of the *Cartesian product* $A \times B$ which can be defined as follows:

$$A \times B = \{X = [X_1, X_2], X_1 \in A, X_2 \in B\} \tag{4}$$

2.2. Computation of reliability interval

Based on the joint BPA and the safety region, the reliability interval $[Bel(G), Pl(G)]$ used to characterize the total degree of belief for the safety $X \in G$ can be calculated as below:

$$Bel(G) = \sum_{C \subseteq G} m_X(C) \tag{5}$$

$$Pl(G) = \sum_{C\cap G \neq \phi} m_X(C) \tag{6}$$

The *belief measure* $Bel(G)$ and *plausibility measure* $Pl(G)$ can be viewed as the lower and upper bounds of the probability measure, which bracket the true probabilistic reliability p_r [33]:

$$Bel(G) \leqslant p_r \leqslant Pl(G) \tag{7}$$

In order to calculate the above two measures, whether $C \subseteq G$ (the focal element C is entirely located inside the safety region G) or $C \cap G \neq \varnothing$ (C is entirely or partially within the region G) should be determined [33]. Therefore, the extreme values of the limit-state function g over each focal element C should be computed:

$$[g_{\min}, g_{\max}] = [\min_{\mathbf{X}\in C} g(\mathbf{X}), \max_{\mathbf{X}\in C} g(\mathbf{X})] \tag{8}$$

To reduce the computational cost, the vertex method [37] can be used to calculate $g_{\min}$ and $g_{\max}$ approximately, in which only the vertex points of each focal element are checked.

Through the above analysis it can be found that two main factors, namely the dimension of the problem and the number of the focal elements for each variable, determine the computational cost of the above reliability analysis. Suppose the dimension of the problem is n and the number of the focal elements for each variable is h, then h^n focal elements in the joint FD will be involved. For each focal element 2^n functional evaluations are required to calculate the extreme values of the limit-state function by using the vertex method, and thereby the total number of functional evaluations for the above reliability analysis will reach $(2h)^n$.