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Grey Wolf Optimizer

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ABSTRACT

This work proposes a new meta-heuristic called Grey Wolf Optimizer (GWO) inspired by grey wolves (Canis lupus). The GWO algorithm mimics the leadership hierarchy and hunting mechanism of grey wolves in nature. Four types of grey wolves such as alpha, beta, delta, and omega are employed for simulating the leadership hierarchy. In addition, the three main steps of hunting, searching for prey, encircling prey, and attacking prey, are implemented. The algorithm is then benchmarked on 29 well-known test functions, and the results are verified by a comparative study with Particle Swarm Optimization (PSO), Gravitational Search Algorithm (GSA), Differential Evolution (DE), Evolutionary Programming (EP), and Evolution Strategy (ES). The results show that the GWO algorithm is able to provide very competitive results compared to these well-known meta-heuristics. The paper also considers solving three classical engineering design problems (tension/compression spring, welded beam, and pressure vessel designs) and presents a real application of the proposed method in the field of optical engineering. The results of the classical engineering design problems and real application prove that the proposed algorithm is applicable to challenging problems with unknown search spaces.

1. Introduction

Meta-heuristic optimization techniques have become very popular over the last two decades. Surprisingly, some of them such as Genetic Algorithm (GA) [1], Ant Colony Optimization (ACO) [2], and Particle Swarm Optimization (PSO) [3] are fairly well-known among not only computer scientists but also scientists from different fields. In addition to the huge number of theoretical works, such optimization techniques have been applied in various fields of study. There is a question here as to why meta-heuristics have become remarkably common. The answer to this question can be summarized into four main reasons: simplicity, flexibility, derivation-free mechanism, and local optima avoidance.

First, meta-heuristics are fairly simple. They have been mostly inspired by very simple concepts. The inspirations are typically related to physical phenomena, animals' behaviors, or evolutionary concepts. The simplicity allows computer scientists to simulate different natural concepts, propose new meta-heuristics, hybridize two or more meta-heuristics, or improve the current meta-heuristics. Moreover, the simplicity assists other scientists to learn meta-heuristics quickly and apply them to their problems.

Second, flexibility refers to the applicability of meta-heuristics to different problems without any special changes in the structure of the algorithm. Meta-heuristics are readily applicable to different problems since they mostly assume problems as black boxes. In other words, only the input(s) and output(s) of a system are important for a meta-heuristic. So, all a designer needs is to know how to represent his/her problem for meta-heuristics.

Third, the majority of meta-heuristics have derivation-free mechanisms. In contrast to gradient-based optimization approaches, meta-heuristics optimize problems stochastically. The optimization process starts with random solution(s), and there is no need to calculate the derivative of search spaces to find the optimum. This makes meta-heuristics highly suitable for real problems with expensive or unknown derivative information.

Finally, meta-heuristics have superior abilities to avoid local optima compared to conventional optimization techniques. This is due to the stochastic nature of meta-heuristics which allow them to avoid stagnation in local solutions and search the entire search space extensively. The search space of real problems is usually unknown and very complex with a massive number of local optima, so meta-heuristics are good options for optimizing these challenging real problems.

The No Free Lunch (NFL) theorem [4] is worth mentioning here. This theorem has logically proved that there is no meta-heuristic best suited for solving all optimization problems. In other words, a particular meta-heuristic may show very promising results on a set of problems, but the same algorithm may show poor performance on a different set of problems. Obviously, NFL makes this field of study highly active which results in enhancing current approaches and proposing new meta-heuristics every year. This also

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motivates our attempts to develop a new meta-heuristic with inspiration from grey wolves.

Generally speaking, meta-heuristics can be divided into two main classes: single-solution-based and population-based. In the former class (Simulated Annealing [5] for instance) the search process starts with one candidate solution. This single candidate solution is then improved over the course of iterations. Population-based meta-heuristics, however, perform the optimization using a set of solutions (population). In this case the search process starts with a random initial population (multiple solutions), and this population is enhanced over the course of iterations. Population-based meta-heuristics have some advantages compared to single solution-based algorithms:

- Multiple candidate solutions share information about the search space which results in sudden jumps toward the promising part of search space.
- Multiple candidate solutions assist each other to avoid locally optimal solutions.
- Population-based meta-heuristics generally have greater exploration compared to single solution-based algorithms.

One of the interesting branches of the population-based meta-heuristics is Swarm Intelligence (SI). The concepts of SI was first proposed in 1993 [6]. According to Bonabeau et al. [1], SI is "*The emergent collective intelligence of groups of simple agents*". The inspirations of SI techniques originate mostly from natural colonies, flock, herds, and schools. Some of the most popular SI techniques are ACO [2], PSO [3], and Artificial Bee Colony (ABC) [7]. A comprehensive literature review of the SI algorithms is provided in the next section. Some of the advantages of SI algorithms are:

- SI algorithms preserve information about the search space over the course of iteration, whereas Evolutionary Algorithms (EA) discard the information of the previous generations.
- SI algorithms often utilize memory to save the best solution obtained so far.
- SI algorithms usually have fewer parameters to adjust.
- SI algorithms have less operators compared to evolutionary approaches (crossover, mutation, elitism, and so on).
- SI algorithms are easy to implement.

Regardless of the differences between the meta-heuristics, a common feature is the division of the search process into two phases: exploration and exploitation [8–12]. The exploration phase refers to the process of investigating the promising area(s) of the search space as broadly as possible. An algorithm needs to have stochastic operators to randomly and globally search the search space in order to support this phase. However, exploitation refers to the local search capability around the promising regions obtained in the exploration phase. Finding a proper balance between these two phases is considered a challenging task due to the stochastic nature of meta-heuristics. This work proposes a new SI technique with inspiration from the social hierarchy and hunting behavior of grey wolf packs. The rest of the paper is organized as follows:

Section 2 presents a literature review of SI techniques. Section 3 outlines the proposed GWO algorithm. The results and discussion of benchmark functions, semi-real problems, and a real application are presented in Sections 4-6, respectively. Finally, Section 7 concludes the work and suggests some directions for future studies.

2. Literature review

Meta-heuristics may be classified into three main classes: evolutionary, physics-based, and SI algorithms. EAs are usually inspired by the concepts of evolution in nature. The most popular algorithm in this branch is GA. This algorithm was proposed by Holland in 1992 [13] and simulates Darwnian evolution concepts. The engineering applications of GA were extensively investigated by Goldberg [14]. Generally speaking, the optimization is done by evolving an initial random solution in EAs. Each new population is created by the combination and mutation of the individuals in the previous generation. Since the best individuals have higher probability of participating in generating the new population, the new population is likely to be better than the previous generation(s). This can guarantee that the initial random population is optimized over the course of generations. Some of the EAs are Differential Evolution (DE) [15], Evolutionary Programing (EP) [16,17], and Evolution Strategy (ES) [18,19], Genetic Programming (GP) [20], and Biogeography-Based Optimizer (BBO) [21].

As an example, the BBO algorithm was first proposed by Simon in 2008 [21]. The basic idea of this algorithm has been inspired by biogeography which refers to the study of biological organisms in terms of geographical distribution (over time and space). The case studies might include different islands, lands, or even continents over decades, centuries, or millennia. In this field of study different ecosystems (habitats or territories) are investigated for finding the relations between different species (habitants) in terms of immigration, emigration, and mutation. The evolution of ecosystems (considering different kinds of species such as predator and prey) over migration and mutation to reach a stable situation was the main inspiration of the BBO algorithm.

The second main branch of meta-heuristics is physics-based techniques. Such optimization algorithms typically mimic physical rules. Some of the most popular algorithms are Gravitational Local Search (GLSA) [22], Big-Bang Big-Crunch (BBBC) [23], Gravitational Search Algorithm (GSA) [24], Charged System Search (CSS) [25], Central Force Optimization (CFO) [26], Artificial Chemical Reaction Optimization Algorithm (ACROA) [27], Black Hole (BH) [28] algorithm, Ray Optimization (RO) [29] algorithm, Small-World Optimization Algorithm (SWOA) [30], Galaxy-based Search Algorithm (GbSA) [31], and Curved Space Optimization (CSO) [32]. The mechanism of these algorithms is different from EAs, in that a random set of search agents communicate and move throughout search space according to physical rules. This movement is implemented, for example, using gravitational force, ray casting, electromagnetic force, inertia force, weights, and so on.

For example, the BBBC algorithm was inspired by the big bang and big crunch theories. The search agents of BBBC are scattered from a point in random directions in a search space according to the principles of the big bang theory. They search randomly and then gather in a final point (the best point obtained so far) according to the principles of the big crunch theory. GSA is another physics-based algorithm. The basic physical theory from which GSA is inspired is Newton's law of universal gravitation. The GSA algorithm performs search by employing a collection of agents that have masses proportional to the value of a fitness function. During iteration, the masses are attracted to each other by the gravitational forces between them. The heavier the mass, the bigger the attractive force. Therefore, the heaviest mass, which is possibly close to the global optimum, attracts the other masses in proportion to their distances.

The third subclass of meta-heuristics is the SI methods. These algorithms mostly mimic the social behavior of swarms, herds, flocks, or schools of creatures in nature. The mechanism is almost similar to physics-based algorithm, but the search agents navigate using the simulated collective and social intelligence of creatures. The most popular SI technique is PSO. The PSO algorithm was proposed by Kennedy and Eberhart [3] and inspired from the social behavior of birds flocking. The PSO algorithm employs multiple particles that chase the position of the best particle and their