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Static and dynamic physically non-linear analysis of concrete structures using a hybrid mixed finite element model

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ABSTRACT

A new hybrid-mixed stress finite element model for the static and dynamic non-linear analysis of concrete structures is presented and discussed in this paper. The main feature of this model is the simultaneous and independent approximation of the stress, the strain and the displacement fields in the domain of each element. The displacements along the static boundary, which is considered to include inter-element boundaries, are also directly approximated. To define the approximation bases in the space domain, complete sets of orthonormal Legendre polynomials are used. The adoption of these functions enables the use of analytical closed form solutions for the computation of all linear structural operators and leads to the development of very effective *p*-refinement procedures. To represent the material *quasi*-brittle behaviour, a physically non-linear model is considered by using damage mechanics. A simple isotropic damage model is adopted and to control strain localisation problems a non-local integral formulation is considered. To solve the dynamic non-linear governing system, a time integration procedure based on the use of the α-HHT method is used. For each time step, the solution of the non-linear governing system is achieved using an iterative algorithm based on a secant method. The model being discussed is applied to the solution of two-dimensional structures. To validate the model, to illustrate its potential and to assess its accuracy and numerical efficiency, several numerical examples are discussed and comparisons are made with solutions provided by experimental tests and with other numerical results obtained using conventional finite elements (CFE).

1. Introduction

It is possible to list three main classes of hybrid and mixed non-conventional finite element formulations [27], namely the hybrid-mixed, the hybrid and the hybrid-Trefftz models [27,29,30,48,49]. All formulations evolve directly from the first principles of mechanics, in particular equilibrium, compatibility and constitutive relations. What distinguishes the three types of formulation is the set of constraints enforced, *a priori*, on the domain approximations. Two models may be derived for each formulation, the displacement and the stress models [40].

In recent years, some of these non-conventional finite element formulations have been extended to the non-linear analysis of concrete structures using isotropic damage models [52]. In [54] the hybrid-mixed stress model based on the use of orthonormal Legendre polynomials is chosen. The stress and the displacement fields in the domain of each element and the displacements on the static boundary are independently approximated. None of the fundamental relations is locally verified and all field equations are enforced in a weighted residual form, ensuring that the discrete numerical model embodies all the relevant properties of the continuum it represents. The Mazars' isotropic model [38] is adopted and a non-local integral formulation where the damage variable is taken as the non-local variable is considered.

In [55] an improved hybrid-mixed stress model is presented and discussed. The approximation of the stress field in the domain is replaced by the approximation of the effective stress field. The isotropic damage models presented by Comi and Perego [16,17] are now adopted using a non-local integral model. An alternative technique based on the definition of an explicit enhanced gradient model has also been tested [52].

The use of hybrid-Trefftz displacement formulations, where the displacements in the domain of each element and the stress field on the kinematic boundary are independently approximated, is reported in [53,56]. The main feature of these models is that the functions used to approximate the displacements are derived from bi-harmonic displacement potentials that solve the Navier equations for a homogeneous elastic material [13,28].

The numerical efficiency of the hybrid-mixed stress model with the independent approximation of the effective stress field and of the hybrid-displacement formulation with Legendre polynomials is assessed in [57]. In this paper, a set of classical benchmark tests

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is chosen to illustrate the use of such models and to assess and compare their numerical performance. Comparisons are also made with solutions obtained with the classical finite element formulation to prove that the proposed models are competitive.

Even if the hybrid-mixed stress model with the independent approximation of the effective stress field and of the hybrid-displacement formulation with Legendre polynomials proved to be robust and computationally competitive when compared with the classical displacement finite element implementations [57], some drawbacks associated to their use can be pointed out. When using hybrid displacement models, local equilibrium, both in the domain and on the static boundary, is enforced in a weak form as in classical finite element formulations. As a consequence, some of the attractive properties associated to the use of non-conventional finite element models may be lost. On the other hand, the use of hybrid-mixed stress models based on the approximation of the effective stress field leads to non-symmetric governing systems that require a much more demanding computational effort for their solution.

To overcome these drawbacks, a new hybrid-mixed stress finite element model, denoted here as 4*f*HMS (*four-field* hybrid-mixed stress model) has been developed and presented [1]. This model considers independent approximations for the stress, strain and displacement fields in the domain of each finite element. The displacement field on the static boundary is also independently approximated.

All of the models previously referred for the material non-linear simulation of softening materials were developed for static analysis. The implementation of hybrid-mixed stress finite element models (the initial versions, with the independent approximation of only two fields in the domain) for the dynamic analysis of linear elastic structures has been reported in [3]. In that work, several classical step-by-step time integration algorithms have been tested together with hybrid-mixed stress models. An alternative approach, based on the definition of a mixed approximation in the time domain [26] was also implemented and assessed.

The main goal of this paper is to present and to discuss the generalisation of the 4*f*HMS model for the physically non-linear dynamic analysis of concrete structures. This model is based on the use of orthonormal Legendre polynomials as approximation functions. The properties of these functions allow for the definition of analytical closed form solutions for the computation of all structural operators in linear analysis. The numerical stability associated to the use of Legendre polynomial bases enables the use of macro-element meshes where the definition of highly effective *p*-adaptive refinement procedures is simplified.

To model concrete's physically non-linear behaviour, two isotropic damage models are considered. The first has been introduced by Comi and Perego [18] and the second corresponds to the classical Mazars model [38]. It must be stressed that the objective of this paper is to present an alternative finite element formulation to model softening materials when the structure under analysis is subjected to cyclic and dynamic loads. No emphasis is given to the damage model to be considered, so only simple isotropic damage models are adopted. The combination of 4*f*HMS models with more sophisticated damage models can be regarded as a future research work.

To control strain localisation problems, a non-local integral formulation is considered [55]. To solve the dynamic non-linear governing system, a time integration procedure based on the use of the α-HHT method [33] is adopted. For each time step, the solution of the non-linear governing system is achieved using an iterative algorithm based on a secant method [20,57].

To validate the model being proposed, to illustrate its potential and to assess its accuracy and numerical efficiency, several numerical examples are discussed and comparisons are made with solutions provided by experimental tests and with other numerical results obtained using conventional finite elements.

This paper is organised as follows: the formulation of the problem and the adopted damage models are presented in Sections 2 and 3. The hybrid-mixed stress finite element model is described in Section 4. The numerical examples are discussed in Section 5 and finally, Section 6 summarises the main conclusions and indicates future research work in this field.

2. Fundamental relations

Consider a set of dynamic forces acting in an elastic body. The equilibrium, compatibility and elasticity equations governing the behaviour of that structure may be expressed as [25]:

$$Ds + b - c_\rho \dot{u} - m_\rho \ddot{u} = 0 \tag{1}$$

$$e = D^* u \tag{2}$$

$$s = ke \tag{3}$$

The vectors s, e and u list the independent components of the stress, strain and displacement fields, respectively. Assuming a geometrically linear behaviour, the differential equilibrium operator, D, and the differential compatibility operator, D^*, represent linear and adjoint operators. Vector b lists the body force components, m_ρ and c_ρ correspond to the mass and damping matrices respectively, and vectors $\dot{u}$ and $\ddot{u}$ collect the velocity and acceleration fields in the domain of the structure. When a physically non-linear behaviour is assumed for the structural material, the constitutive relations may be expressed in the following generic format:

$$s = s(e) \tag{4}$$

The boundary of the structure may be subdivided into two complementary regions: the kinematic boundary, Γ_u, in which the value for the displacement field is prescribed and the static boundary, Γ_t, where the applied forces are prescribed. It can be written:

$$u = u_\Gamma \text{ on } \Gamma_u \tag{5}$$

$$Ns = t \text{ on } \Gamma_t \tag{6}$$

The vectors u_Γ and t gather the components of the prescribed displacements and forces on the kinematic and static boundaries, respectively. Matrix N lists the components of the unit outward normal vector.

3. Damage models

The first isotropic damage model adopted in this work is based on the free energy density potential defined in Eq. (7), where k corresponds to the elastic tensor of the virgin material and d is the isotropic scalar damage variable which varies between zero (virgin material) and one (material totally damaged).

$$\begin{aligned}\Psi &= \frac{1}{2}(1-d)\{\varepsilon\}^t[k]\{\varepsilon\} + \Psi_{\text{int}}(\xi) \\ &= \frac{1}{2}(1-d)\{\varepsilon\}^t[k]\{\varepsilon\} + k(1-\xi)\sum_{i=0}^{n}\frac{n!}{i!}\ln^i\left(\frac{c}{1-\xi}\right)\end{aligned} \tag{7}$$

Using this state potential, it is possible to obtain the following state equations:

$$\begin{cases} \{\sigma\} = \frac{\partial\Psi}{\partial\varepsilon} = (1-d)[k]\{\varepsilon\} \\ \chi = \frac{\partial\Psi}{\partial\xi} = k\ln^n\left(\frac{c}{1-\xi}\right) \\ Y = -\frac{\partial\Psi}{\partial d} = \frac{1}{2}\{\varepsilon\}^t[k]\{\varepsilon\} \end{cases} \tag{8}$$