Advances in Engineering Software 65 (2013) 158–167

Contents lists available at SciVerse ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

As-rigid-as-possible mesh deformation and its application in hexahedral mesh generation

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ARTICLE INFO

Article history:
Received 10 April 2013
Received in revised form 4 June 2013
Accepted 30 June 2013
Available online 20 July 2013

Keywords:
Mesh deformation
Scale-invariant intrinsic variables
Elastic deformation energy
Hexahedral mesh generation
As-rigid-as-possible
Quadric energy function

ABSTRACT

This paper presents an efficient and stable as-rigid-as-possible mesh deformation algorithm for planar shape deformation and hexahedral mesh generation. The deformation algorithm aims to preserve two local geometric properties: scale-invariant intrinsic variables and elastic deformation energy, which are together represented in a quadric energy function. To preserve these properties, the position of each vertex is further adjusted by iteratively minimizing this quadric energy function to meet the position constraint of the controlling points. Experimental results show that the deformation algorithm is efficient, and can obtain physically plausible results, which have the same topology structure with the original mesh. Such a mesh deformation method is useful to project the source surface mesh onto the target surfaces in hexahedral mesh generation based on sweep method, and application results show that the proposed method is feasible to mesh projection not only between similar surface contours but also dissimilar surface contours.

1. Introduction

Mesh has gradually become the mainstream representation of geometric models, and deformation technique for planar mesh model has received a lot of attentions in recent years. Planar mesh deformation is widely used in many application fields, such as computer aided design, mesh generation, shape modeling, computer animation and other applications. A good mesh deformation algorithm aims to produce naturally deforming results, which are homeomorphic to the original planar mesh, and the final positions of the controlling points should be precise. Furthermore, in the field of solid mesh generation, hexahedral mesh generation has always been concerned, since hexahedral mesh offers several numerical advantages over tetrahedral mesh due to its tensor product nature. Surveys show that more than 60% volumes are meshed by sweep method, which shows that sweep method has been the workhorse algorithm in hexahedral mesh applications [1]. Given a swept volume, it is necessary to project the source surface mesh onto the target surfaces, and therefore the most challenging issue to be dealt with by any sweep method is the interpolation between the source mesh contours and the target surface contours.

In this paper, we focus on planar mesh deformation, a critical step towards generation of hexahedral mesh based on sweep method, and propose a new as-rigid-as-possible planar mesh deformation algorithm. The deformation algorithm aims to preserve two local properties: scale-invariant intrinsic variables and elastic deformation energy, which are together represented in a quadric energy function. We iteratively minimize the linear quadric energy function to adjust the position of each vertex to obtain a physically plausible deforming result, which can meet the position constraints of the controlling points. We also introduce a scheme to project the source surface mesh onto the target surfaces in sweep based hexahedral mesh generation via the proposed deformation algorithm, and numerical experiments show that a homeomorphous deformed mesh can be obtained and used to achieve the projection between surfaces with dissimilar contours.

2. Related work

The problem of shape deformation for planar mesh has attracted huge interest in the past, which started with the work of Sederberg and Parry [2], who produced deforming results by the most well-known method, called free form deformation (FFD). FFD is a space-based technique [3,4], which embeds a shape into a lattice and then makes the shape to deform by moving the controlling points of the lattice. While FFD method is applicable to any type of shapes, easy to implement and very efficient in computation, it cannot keep the geometric features of the input shapes accurately. For shapes with significantly skeleton structure,

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http://dx.doi.org/10.1016/j.advengsoft.2013.06.020

skeleton-based deformation [5] provides an intuitive tool to control the deformation of these shapes, and it is convenient for users to manipulate the skeletons. The most important issue to skeleton-based deformation method is to determine the weight of each vertex in the affected mesh area, and the deformed results are quite sensitive to weight selection, which is a painful process for users.

To obtain a physically deforming result, Gibson and Mirtich [6] proposed a physically based shape deformation algorithm by mass–spring system, which is very easy to implement. However, it is too slow to converge and not stable in some cases, and various parameters need to be carefully adjusted. Celniker and Gossard [7] provide a more accurate simulation tool to achieve physically plausible results by using finite element theory, but it is complex and very expensive in computation, making it unsuitable for interactive deformation applications. Alexa et al. [8] presented as-rigid-as-possible shape deformation method first, and more methods [9,10] were developed by using a similar idea. The key contribution of the idea is a local to global algorithm, which combines a local rigid transformation on each triangle, with a global stitch operation to all triangles. But these methods are unstable in some cases.

Recently, many feature preserving deformation methods are proposed for planar mesh models. They try to minimize an energy function representing local properties of the model, and the most important thing to these methods is the selection of local properties. The more related feature preserving approaches proposed in this paper are those works introduced by Igarashi et al. [10], Weng et al. [11], and Guo et al. [12]. Igarshi et al. proposed a two-step algorithm to deform a planar shape by manipulating a few controlling points. The first step finds a local rigid transformation for each triangle and the second step adjusts its scale globally. The key contribution is to use quadratic error metrics so that each minimization problem can be solved quickly and stably. Duo to its linear nature, the two-step algorithm may cause implausible results in some cases. Weng et al. presented a 2D shape deformation algorithm using non-linear least squares method. Their method aims to preserve the Laplacian coordinates of the boundary curve and the local area of the shape interior, and a physically plausible deformed result can be obtained by minimize a nonlinear energy function using an iterative Gauss–Newton method. This non-linear 2D shape deformation does not take into account the difference of mesh structure, and Guo et al. provide a detailed analysis of the shape deformation method and prove that triangle mesh gains more advantages in deforming. Based on the triangle mesh, they proposed an improved edge length preserving shape deformation algorithm, which is enough to preserve the local, global and boundary properties of the shape. For some deformations with huge shape changes, the aforementioned methods may produce implausible results with face flips. Differential domain techniques [13–15] cast deformation as an energy minimization problem, and it preserves surface details and produces visually pleasing deformation results by distributing errors globally through least-squares minimization. Sorkine and Alexa [16] proposed an ARAP surface deformation method by using an iterative minimizing scheme, which can make small parts of the model change as rigidly as possible.

Sweep based method is the most common used method in hexahedral mesh generation, and the main issue to be dealt with by any sweep algorithm is the projection of the source surface mesh onto the target surfaces. Goodrich [17] proposed an orthogonal projection method to map nodes onto the target surfaces, the projection process is named as root finding, and the root finding problem is time-consuming and very complex. Knupp [18] presented a node placement way based on the linear transformation between bounding node loops and smoothing method. Roca and Sarrate [19,20] presented a method such that the projection of the source mesh onto the target surfaces is determined by means of an improved least-square approximation of an affine mapping. The affine mapping is a linear transformation that preserves the "straightness" of shape, the relative position of each node keeps unchanged. Hence, the affine mapping method is not suitable for the projection between surfaces with dissimilar contours.

3. Planar mesh deformation

Mesh has been widely used in shape representation, and shapes discussed in our algorithm are composed of triangle meshes and quadrilateral meshes. Our algorithm can be viewed as a feature preserving deformation method, which aims to preserve the scale-invariant intrinsic variables and elastic deformation energy of planar mesh. The scale-invariant intrinsic variables represent the local intrinsic geometry characteristic of planar mesh and they are invariant to geometric transformation such as translation, rotation and scaling. While preserving scale invariant intrinsic variables often produces good deformation results for slight deformations, it is not enough to produce physically plausible deformation results for large deformations with too much controlling points. In these situations, the deformed mesh with local self-intersection is not homeomorphic to the original planar mesh. Therefore, we introduce the elastic deformation energy to preserve the area of each mesh and to ensure the validity of deformed mesh. Instead of minimizing a non-quadratic energy function, the proposed algorithm deals only with quadratic energy function that consists of three parts: Scale-invariant intrinsic variables preserving, elastic deformation energy and position constraints of all controlling points, and a linear solution can be used iteratively to obtain a homeomorphous deformed mesh.

We consider a two-dimensional mesh $M = (V, E)$ is an abstract simplicial complex representing the connectivity of the mesh, where $V = \{v_0, v_1, \ldots, v_{n-1}\}$is the set of vertices, n is the number of vertices; E is the simplicial complex that contains vertices $v = \{i\} \in E$, edges $e = \{i,j\} \in E$ and faces $f = \{i,j,k\} \in E$. The adjacent vertices of vertex v_i are denoted by $N(v_i) = \{j|(i,j) \in E\}$, the number of adjacent vertices is called the degree of vertex v_i, and is denoted as $|N(v_i)|$.

3.1. Scale-invariant intrinsic variables

The scale-invariant intrinsic variables are defined for adjacent edges of vertices at its first order neighborhood, and it is composed of the length ratio and orientation angle of adjacent edges [21]. For a vertex v_0 in a planar mesh M and the polygon constructed by its first order neighborhood (see Fig. 1), the adjacent edge of v_0v_i is v_0v_{i+1} in counterclockwise; α_i is the orientation angle between v_0v_i and v_0v_{i+1}; $\lambda_i = ||v_0v_{i+1}||/||v_0v_i||$ is the length ratio between v_0v_i and v_0v_{i+1}. In order to describe the scale-invariant intrinsic

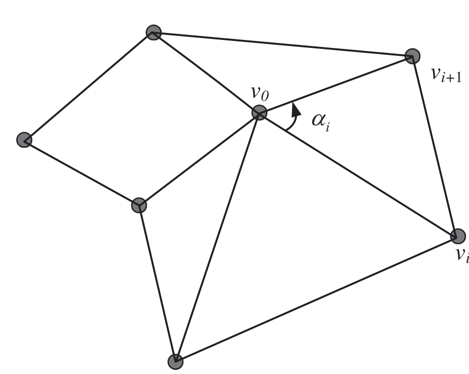

Fig. 1. First order neighborhood of vertex.