Advances in Engineering Software 41 (2010) 442–450

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

# Cost optimization of industrial steel building structures

S. Kravanja *, T. Žula

University of Maribor, Faculty of Civil Engineering, Smetanova 17, 2000 Maribor, Slovenia

ARTICLE INFO

Article history:
Available online 17 April 2009

Keywords:
Cost optimization
Structural optimization
Topology optimization
Cross-section optimization
Mixed-integer non-linear programming
MINLP
Industrial building
Steel structures

ABSTRACT

The paper presents the simultaneous cost, topology and standard cross-section optimization of single-storey industrial steel building structures. The considered structures are consisted from main portal frames, which are mutually connected with purlins. The optimization is performed by the mixed-integer non-linear programming approach, MINLP. The MINLP superstructure of different structure/topology and standard cross-section alternatives has been generated and the MINLP optimization model of the structure has been developed. The defined cost objective function is subjected to the set of (in)equality constraints known from the structural analysis. Internal forces and deflections are calculated by the elastic first-order analysis constraints. The dimensioning constraints of steel members are defined in accordance with Eurocode 3. The modified outer-approximation/equality-relaxation (OA/ER) algorithm, a two-phase MINLP strategy and a special prescreening procedure of discrete alternatives are used for the optimization. A numerical example of the cost optimization of a single-storey industrial steel building is presented at the end of the paper.



## 1. Introduction

Structural engineers and designers are in the daily engineering praxis required to design the cheapest possible structures with the minimum amount of used material and technical equipment. The use of modern optimization methods thus becomes a great opportunity in the area of structural engineering.

Single-storey industrial steel building structures are probably the most frequently built type of structures among various skeletal framed steel constructions. Many different optimization approaches have been proposed in the near past for the optimization of these structures. E.g. Lee and Knapton [1] have performed a constrained non-linear cost optimization of steel portal framed building. O'Brien and Dixon [2] have proposed a linear programming approach for the optimal design of pitched roof frames. Gurlement et al. [3] have introduced a practical method for single-storey steel structures, based on a discrete minimum weight design and Eurocode design constraints. Saka [4] has considered an optimum design of pitched roof steel frames with haunched rafters by using a genetic algorithm. Kamal et al. [5] have carried out a weight optimization of two-hinged steel portal frames under multiple loadings. One of the latest researches reported in this field is the work of Hernández et al. [6], where authors have considered minimum weight design of steel portal frames with software developed for structural optimization.

This paper deals with the simultaneous cost, topology and standard cross-section optimization of single-storey industrial steel building structures. The considered building structures are consisted from main portal frames, which are mutually connected with purlins. The task of the optimization is to find the minimal structure's material and labour costs, the optimal topology with the optimal number of portal frames and purlins as well as the optimal standard cross-sections of steel members.

The optimization is performed by the mixed-integer non-linear programming (MINLP). The MINLP is a combined discrete and continuous optimization technique. It handles with continuous and discrete binary 0–1 variables simultaneously. While continuous variables are defined for the continuous optimization of parameters (dimensions, stresses, strains, weights, costs, etc.), discrete variables are used to express discrete decisions (topology and standard cross-section alternatives). Since continuous and discrete optimizations are carried out simultaneously, the MINLP approach also finds optimal continuous parameters (e.g. structural costs), structural topology and discrete standard sizes simultaneously.

The MINLP discrete/continuous optimization problems of such framed building structures are in most cases comprehensive, non-convex and highly non-linear. The optimization requires the generation of the building's MINLP superstructure of different topology and standard cross-section alternatives and the development of the MINLP optimization model. Since the objective of the optimization is to minimize the structure's self-manufacturing costs, the cost objective function has been defined. It comprises the material, fabrication and anti-corrosion protection painting costs as well as the assembling and erection costs of the structure.

* Corresponding author. Tel.: +386 2 2294 300; fax: +386 2 2524 179.
E-mail addresses: stojan.kravanja@uni-mb.si (S. Kravanja), tomaz.zula@uni-mb.si (T. Žula).

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doi:10.1016/j.advengsoft.2009.03.005

The cost objective function is subjected to the set of equality and inequality constraints known from the structural analysis. Internal forces and deflections are calculated by the elastic first-order analysis constraints. The dimensioning constraints of steel members are defined in accordance with Eurocode 3 [7] for the conditions of both the ultimate and serviceability limit states.

The modified outer-approximation/equality-relaxation algorithm is used to perform the optimization, see Kravanja and Grossmann [8], Kravanja et al. [9–11]. The two-phase MINLP optimization is proposed. It starts with the topology optimization, while the standard dimensions are temporarily relaxed into continuous parameters. When the optimal topology is found, the standard dimensions of the cross-sections are re-established and the simultaneous discrete topology and standard dimension optimization of the beams, columns and purlins is then continued until the optimal solution is found. In order to reduce a high number of structure alternatives and enable a normal solution of the MINLP, a special prescreening procedure has been developed, which automatically reduces the binary variables of alternatives into a reasonable number. The optimization at the second phase includes only those 0–1 variables which determine the topology and standard dimension alternatives close to the values, obtained at the first MINLP optimization phase.

## 2. MINLP model formulation

The MINLP optimization of the industrial steel building needs the generation of the building's MINLP superstructure, which is composed of various topology and discrete design alternatives that are all candidates for a feasible and optimal solution. While topology alternatives represent different selections and interconnections of corresponding structural elements – portal frames and purlins, discrete design alternatives include different standard cross-sections of columns, beams and purlins. The MINLP superstructure is modeled according to the MINLP model formulation.

It is assumed that a general non-convex and non-linear discrete/continuous optimization problem can be formulated as an MINLP problem in the form:

$$\begin{aligned} \min \quad & z = \mathbf{c}^{\mathrm{T}}\boldsymbol{y} + f(\boldsymbol{x}) \\ \text{s.t.} \quad & \boldsymbol{h}(\boldsymbol{x}) = \mathbf{0} \\ & \mathbf{g}(\boldsymbol{x}) \leq \mathbf{0} \\ & \boldsymbol{By} + \boldsymbol{Cx} \leq \boldsymbol{b} \\ & \boldsymbol{x} \in X = \{\boldsymbol{x} \in R^n : \boldsymbol{x}^{\mathrm{LO}} \leq \boldsymbol{x} \leq \boldsymbol{x}^{\mathrm{UP}}\} \\ & \boldsymbol{y} \in Y = \{0,1\}^m \end{aligned} \tag{MINLP}$$

where $\boldsymbol{x}$ is a vector of continuous variables specified in the compact set $X$ and $\boldsymbol{y}$ is a vector of discrete, binary 0–1 variables. Functions $f(\boldsymbol{x})$, $\boldsymbol{h}(\boldsymbol{x})$ and $\mathbf{g}(\boldsymbol{x})$ are non-linear functions involved in the objective function $z$, equality and inequality constraints, respectively. All functions $f(\boldsymbol{x})$, $\boldsymbol{h}(\boldsymbol{x})$ and $\mathbf{g}(\boldsymbol{x})$ must be continuous and differentiable.

In the context of structural optimization, continuous variables $\boldsymbol{x}$ define structural parameters (actions, dimensions, stresses, deflections, costs,...) and binary variables $\boldsymbol{y}$ represent the potential existence of structural elements within the defined superstructure. An extra binary variable $y$ is assigned to each structural element. The element (the portal frame or purlin) is then selected to compose the structure if its subjected binary variable takes value one ($y = 1$), otherwise it is rejected ($y = 0$). Binary variables also define the choice of discrete/standard cross-sections.

The economical objective function $z$ involves fixed costs in the term $\boldsymbol{c}^{\mathrm{T}}\boldsymbol{y}$, while the dimension dependant costs are included in the function $f(\boldsymbol{x})$. Non-linear equality and inequality constraints $\boldsymbol{h}(\boldsymbol{x}) = \mathbf{0}$, $\mathbf{g}(\boldsymbol{x}) \leqslant \mathbf{0}$ and the bounds of the continuous variables represent the rigorous system of the design, loading, resistance, stress, deflection, etc. constraints known from the structural analysis. Logical constraints that must be fulfilled for discrete decisions and structure configurations, which are selected from within the superstructure, are given by $\boldsymbol{By} + \boldsymbol{Cx} \leqslant \boldsymbol{b}$. These constraints describe relations between binary variables, restore interconnection relations between currently selected or existing structural elements (corresponding $y = 1$) and cancel relations for currently rejected or nonexisting elements (corresponding $y = 0$), define continuous design variables for each existing structural element and define the structural topology and standard cross-sections of elements. It should be noted, that the comprehensive MINLP model formulation for mechanical structures may be found elsewhere, see Kravanja et al. [12].

## 3. Optimization model

The single-storey industrial steel building structure is consisted from equal main portal frames, mutually connected with equal purlins, see Fig. 1. Each the portal frame is constructed from two columns and two beams. Purlins run continuously over the portal frames. Columns, beams and purlins are proposed to be built up from steel standard hot rolled European wide flange I sections (HEA sections), see Fig. 2. The global building geometry (including the frame span $L_f$, the building length $L_{\mathrm{TOT}}$, the column height $H_C$ and the overhight $f$) is proposed to be fix through the optimization. The vertical and horizontal bracing systems as well as the wall sheeting rails are not included in this optimization.

On the basis of the mentioned MINLP model formulation, the MINLP optimization model Single-Storey Industrial Steel Building OPTimization (SSISBOPT) has been developed for the cost optimization of the industrial steel building structures. As an interface for mathematical modeling and data inputs/outputs general algebraic modeling system (GAMS), a high level language by Brooke et al. [13] is used.

The optimization model comprises input data, continuous and discrete binary variables, the structure's cost objective function, structural analysis constraints and logical constraints. The cost objective function is subjected to the set of (non)linear structural analysis constraints and linear logical constraints.

*Input data* comprises sets for the topology and cross-section alternatives, scalars and parameters. Defined are $m$, $m \in M$, number of purlins; $n$, $n \in N$, number of portal frames; $i$, $i \in I$; $j$, $j \in J$; and $k$, $k \in K$, standard cross-section alternatives for columns, beams and purlins separately.

*Scalars* in input data include the industrial building global geometry: the frame span $L_f$, the length of the industrial building $L_{\mathrm{TOT}}$, the height of the column $H_C$ and the overheight of the frame beam $f$. The yield strength of structural steel $f_y$, the elastic modulus of steel $E$, the shear modulus of steel $G$, the density of steel $\rho$, the mass of the roof $g_r$, snow $s$, the vertical wind $w_v$, the horizontal wind $w_h$, the partial safety factor for permanent load $\gamma_g$ (1.35), the partial safety factor for variable load $\gamma_q$ (1.50), the resistance partial safety factors $\gamma_{M0}$ (1.10) and $\gamma_{M1}$ (1.10), the price of the structural steel $C_{\mathrm{mat}}$, the price of the anti-corrosion and fire protection painting $C_{paint}$, the erection price of the portal frame $C_{erect,frame}$, the erection price of the purlin $C_{erect,purlin}$, the coefficient for calculating the fabrication costs $C_{fabr}$, etc. are defined as input data.

*Parameters* in input data comprise the vectors of different discrete alternative constants, e.g. $\boldsymbol{q}_i^{A_C}$, a vector of $i$, $i \in I$, discrete standard cross-section area alternatives for columns; $\boldsymbol{q}_j^{A_B}$, a vector of $j$, $j \in J$, discrete standard cross-section area alternatives for beams; and $\boldsymbol{q}_k^{A_P}$, a vector of $k$, $k \in K$, discrete standard cross-section area alternatives for purlins. Similarly are defined all other cross-section constants for heights, breadths, the web and flange thickness, the second moment of areas, the warping constants, etc.