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An integration scheme for electromagnetic scattering using plane wave edge elements

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ABSTRACT

Finite element techniques for the simulation of electromagnetic wave propagation are, like all conventional element based approaches for wave problems, limited by the ability of the polynomial basis to capture the sinusoidal nature of the solution. The Partition of Unity Method (PUM) has recently been applied successfully, in finite and boundary element algorithms, to wave propagation. In this paper, we apply the PUM approach to the edge finite elements in the solution of Maxwell's equations. The electric field is expanded in a set of plane waves, the amplitudes of which become the unknowns, allowing each element to span a region containing multiple wavelengths. However, it is well known that, with PUM enrichment, the burden of computation shifts from the solver to the evaluation of oscillatory integrals during matrix assembly. A full electromagnetic scattering problem is not simulated or solved in this paper. This paper is an addition to the work of Ledger and concentrates on efficient methods of evaluating the oscillatory integrals that arise. A semi-analytical scheme of the Filon character is presented.

1. Introduction

The problem of computing the electromagnetic field scattered by a body when subjected to an incident wave has practical importance in radar cross section prediction. There are several numerical methods that have been proposed for the simulation of these fields, for example the discontinuous Galerkin method [1] is attracting much attention. Node based finite elements impose a continuity of all components of the field across inter-element boundaries which is not a necessary property of the field. Using such node based finite elements may lead to the occurrence of spurious modes in the numerical solutions. Edge elements, first introduced by Nédélec [2–4], assign degrees of freedom to the edges rather than to the nodes of the elements and have the property that they ensure the continuity of the tangential component of the field across inter-element boundaries while allowing for jumps in the normal component of the field. Edge elements are free of spurious modes.

For the lowest order edge element, the tangential component of the solution is constant on each edge. Consequently the accuracy achieved by this element is low, leading to a requirement for very dense meshes. Compatible, arbitrary order, quadrilateral and triangular edge elements have been developed by Demkowicz and Rachowicz [5] and Ainsworth and Coyle [6]; the latter have been shown to have better conditioning properties and thus will be used here.

The partition of unity method (PUM) developed by Melenk and Babuška [7,8] is a general numerical approximation technique in which the approximation space is enriched by the inclusion of a set of analytical functions known to form a basis for the solution. The approach has been implemented in a finite element scheme under the heading of the Partition of Unity Finite Element Method (PUFEM). The motivation for the use of these new elements is to escape the limitations of conventional finite element procedures, which for wave problems impose an upper bound on nodal spacing of around 10% of the wavelength under consideration. This has led to a rule of thumb being adopted requiring a minimum of around 10 degrees of freedom per wavelength to be used in conventional finite element models. In PUFEM the field is expanded in a discrete series of plane waves, each propagating at a specified angle. These angles can be uniformly distributed or may be carefully chosen. This expansion allows each element to span many wavelengths.

For the simulation of wave phenomena, the PUFEM has been applied to a range of Helmholtz wave diffraction problems in the frequency domain. Laghrouche et al. [9] showed that (when compared with conventional fine finite element meshes) the number of variables could be reduced by up to 96%. Mayer and Mandel [10] presented a similar method with the name Finite Ray Element Method. Farhat et al. [11–13] have proposed the Discontinuous Enrichment Method in which the standard finite element polynomial field is enriched by plane waves. Ortiz and Sanchez [14] have

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developed a three-node wave finite element based on the partition of unity model. Strouboulis et al. [15] apply the approach on Cartesian meshes in the generalised finite element method and analyse the convergence properties.

Plane waves were used in the approximation of integral equations in electromagnetic scattering by de La Bourdonnaye [16] under the name of Microlocal Discretisation. Further experiences in implementing the PUM in boundary element approximations (PUBEM) are reported by Perrey-Debain et al. [17–19]. Trevelyan et al. [20] presented a successful adaptive PUBEM scheme in which wave directions are added in response to an error indicator. A particularly effective strategy for integral equation approximations can be found in the enrichment of the approximation space using a single plane wave, i.e. the incident wave [21–23]. This offers the benefit of a solution complexity independent of the wave number, enabling the analysis of scatterers of size $10^6\lambda$, but is limited to scattering by convex obstacles that are large with respect to the wavelength. Electromagnetic scattering using plane wave enrichment of Ainsworth and Coyle [6] edge elements has previously been presented by Ledger et al. [24,25]. This paper is an addition to that work.

Most authors introducing the PUM for the solution of wave problems have reported ill-conditioning problems with these methods. It is noted that these problems can be ameliorated if the system is not over-defined, i.e. if the number of degrees of freedom is not too great [18]. A detailed study of the conditioning of PUFEM approximations using edge elements is a subject for future research. In this paper we focus on efficient methods of evaluating the oscillatory integrals that arise in the matrix entries.

The reduction in the number of active variables by using such a plane wave basis comes at the price of some computationally intensive numerical integration over the elements. Since a single element might contain many wavelengths, the integrands to be evaluated in order to determine terms in the governing matrices become highly oscillatory. It is well known that polynomial representations of trigonometrical functions are not accurate and are expensive to compute. Therefore, conventional Gauss–Legendre integration requires a very large number of integration points. Computational integration of oscillatory functions is currently an active area of research. Readers are referred to the semi-analytical integration rules of Bettess et al. [26], for example, and other works by Iserles et al. [27], Huybrechs and Vandewalle [28,29], Langdon and Chandler-Wilde [30], Honnor and Trevelyan [31].

2. Mathematical model of electromagnetics

Electromagnetic phenomena are governed by Maxwell's equations, which for harmonically oscillating functions with a single frequency, ω, lead to the vector wave equation. The vector wave equation can be expressed either in terms of electric field intensity, $\underset{\sim}{E}$, or in terms of magnetic field intensity, $\underset{\sim}{H}$. The vector wave equation for the electric field is [32,33]

$$\nabla \times (\mu^{-1}\nabla \times \underset{\sim}{E}) - \omega^2 \varepsilon \underset{\sim}{E} = -\mathrm{i}\omega \underset{\sim}{J} \tag{1}$$

where μ is the permeability, ε is the permittivity, and $\underset{\sim}{J}$ is the electric current density. The vector wave equation for the magnetic field is [32,33]

$$\nabla \times (\varepsilon^{-1}\nabla \times \underset{\sim}{H}) - \omega^2 \mu \underset{\sim}{H} = \nabla \times (\varepsilon^{-1} \underset{\sim}{J}) \tag{2}$$

The vector wave equations (1) or (2), is used in conjunction with the continuity equation [32,33]

$$\nabla \cdot \underset{\sim}{J} = -\mathrm{i}\omega\rho \tag{3}$$

where ρ is the electric charge density. In this work we assume that the conductivity of the medium is negligible and the permittivity, ε, and permeability, μ, are unity. This does not lead to any loss in generality since for homogeneous media they are scalar constants. For inhomogeneous media they become functions of position.

3. Geometry representation and basis functions

The edge element basis cannot be used to represent the geometry. Instead, traditional nodal methods must be used. Where the order of approximation of the field variable is increased and the mesh kept constant, the use of a linear sub-parametric discretisation can result in inaccurate solutions. In this work we use the linear blending function approach [34]. There is no requirement for additional nodes on boundary edges of the element and the curve of the boundary edges exactly follows the true curve of the boundary. In this section we describe the hierarchic compatible arbitrary order quadrilateral and triangular edge elements of Ainsworth and Coyle [6].

3.1. Quadrilateral edge element

The master quadrilateral element is shown in Fig. 1. For an element of order p, the variation of the electric field over this element is given in the interpolated form

$$\underset{\sim}{U}(\xi_1,\xi_2) = \sum_{i=1}^{4}\sum_{j=0}^{p} u_j^i \phi_j^i + \sum_{j=0}^{p}\sum_{k=1}^{p} u_{j,k}^{I_{\xi_1}} \phi_{j,k}^{I_{\xi_1}} + \sum_{j=0}^{p}\sum_{k=1}^{p} u_{j,k}^{I_{\xi_2}} \phi_{j,k}^{I_{\xi_2}} \tag{4}$$

where $\underset{\sim}{U}$ denotes either $\underset{\sim}{E}$ for transverse electric polarisation or $\underset{\sim}{H}$ for transverse magnetic polarisation, $\underset{\sim}{\phi}$ are the vector shape functions and u are the unknowns. The basis functions associated with the element edges, ϕ_j^i, and interior basis functions, $\phi_{j,k}^{I_{\xi_1}}$ and $\phi_{j,k}^{I_{\xi_2}}$, are piecewise polynomials constructed from shape functions, edge vectors, Legendre polynomials and integrated Legendre polynomials as defined in [6].

3.2. Triangular edge element

The master triangular element is shown in Fig. 2. For an element of order p, the variation of the electric field over this element is

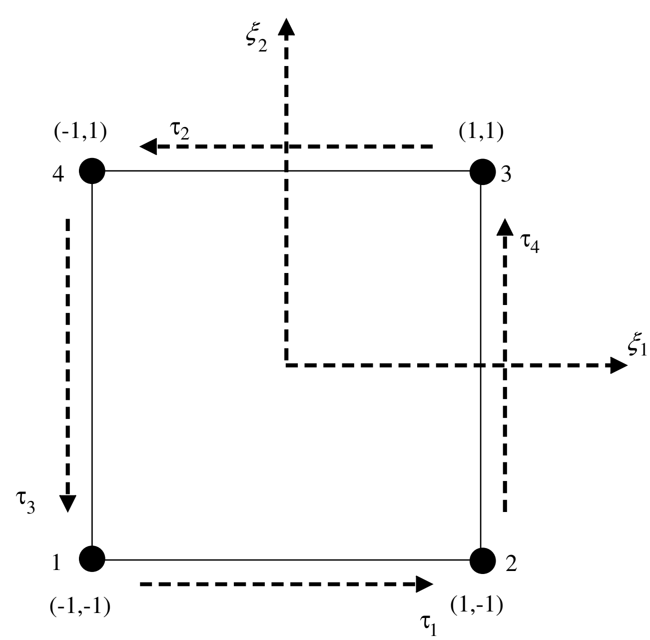

Fig. 1. The quadrilateral master element.