Advances in Engineering Software 92 (2016) 40–47

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

Precise trajectory optimization for articulated wheeled vehicles in cluttered environments

Bai Li[a], Zhijiang Shao[a,b,*]

[a] College of Control Science & Engineering, Zhejiang University, Hangzhou 310027, China
[b] State Key Laboratory of Industrial Control Technology, Hangzhou, PR China

ARTICLE INFO

Article history:
Received 4 June 2015
Revised 18 October 2015
Accepted 25 October 2015
Available online 22 November 2015

Keywords:
Articulated vehicle
Trajectory planning
Mesh generation
Collision avoidance
Large-scale optimization
Computational guidance and control

ABSTRACT

Trajectory planning refers to planning a time-dependent path connecting the initial and final configurations with some special constraints simultaneously considered. It is a critical aspect in autonomously driving an articulated vehicle. In this paper, trajectory planning is formulated as a dynamic optimization problem that contains kinematic differential equations, mechanical/environmental constraints, boundary conditions and an optimization objective. The prevailing numerical methods for solving the formulated dynamic optimization problem commonly disregard the constraint satisfactions between every two adjacent discretized mesh points, thus resulting in failure when the planned motions are actually implemented. As a remedy for this limitation, the concept of minute mesh grid is proposed, which improves the constraint satisfactions between adjacent rough mesh points. On the basis of accurate penalty functions, large-scale constraints are successfully incorporated into the optimization criterion, thus transforming the dynamic optimization problem into a static one with simple bounds on the decision variables. Simulation results verify that our proposed methodology can provide accurate results and can deal with various optimization objectives uniformly.

1. Introduction

An articulated vehicle is a vehicle with a permanent or semi-permanent pivoting joint in its structure [1]. In a broad sense, any vehicle towing a trailer can be considered as articulated. Compared with a rigid-body vehicle in the same length, an articulated vehicle turns with a significantly smaller turning radius [2]. Also, articulation aids the vehicle in keeping contact with the rough terrain. Both merits have motivated the application of articulated vehicles, including buses, trams, trains, trucks and robotic floor cleaner [3]. This current study focuses on the trajectory generation issue of articulated wheeled vehicles.

Trajectory generation involves planning a time-dependent path connecting the initial and desired final configurations with simultaneous consideration of some predefined requirements [4,5]. Viale et al. [6] proposed a practical multi-step trajectory planner that calculates geometric paths first and then generates smooth trajectories. Although calculating the preliminary geometric paths (consisting of line segments and circular arcs) is automated and fast, the overall method is ineffective in handling intricate scenarios directly and precisely, particularly schemes with time-dependent constraints and irregularly placed obstacles. Other geometric-based methods or first-path-then-trajectory methods (e.g., [7–11]) also suffer from this limitation. Wang and Cartmell [12] adopted a function-fitting approach to calculate time-dependent profiles directly. Unfortunately, collision avoidance was not considered in their study. Zare et al. [13] developed a fuzzy-based method, wherein three separate fuzzy controllers are used for forward maneuvers, destination approaching and collision avoidance. However, the kinematic model and environment are not precisely described in their works. More importantly, fuzzy-based methods are generally adopted to determine feasible rather than optimal or optimized trajectories.

In addition to the aforementioned trajectory planners, previous studies incorporate part of trajectory generation with trajectory tracking. For example, a path is planned first and then trajectory tracking is directly done. Trajectory tracking involves closed-loop execution. Thus control theories are utilized. In literature, backward motions are particularly treated because of control instability [14–17]. A well-known issue associated with backward motion is jackknifing [18]. Theoretically, given that the entire articulated system is physically linked, the velocity vector components along the links should be consistent. Fig. 1(a) shows an example with such consistency, whereas Fig. 1(b) shows an example with jackknifing owing to the inconsistency. When jackknifing occurs, the link will be damaged or the trailer will sideslip. To avoid jackknifing, a small-angle constraint was proposed, which requires that the error between the

---

[*] Corresponding author. Tel.: +86 571 87953068.
E-mail addresses: libai@zju.edu.cn, libaioutstanding@163.com (B. Li), szj@zju.edu.cn (Z. Shao).

http://dx.doi.org/10.1016/j.advengsoft.2015.10.008
0965-9978/

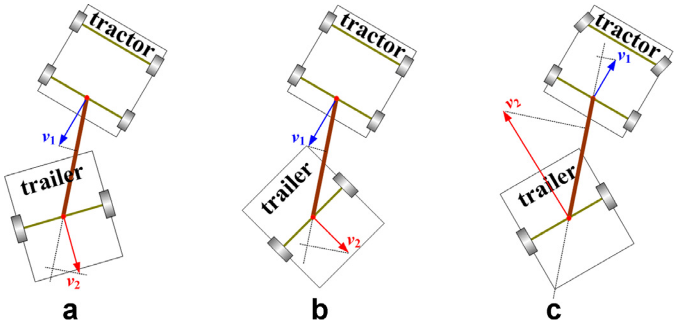

Fig. 1. Schematics regarding jackknifing phenomenon: (a) normal example; (b) backward movement with jackknifing; (c) forward movement example with jackknifing.

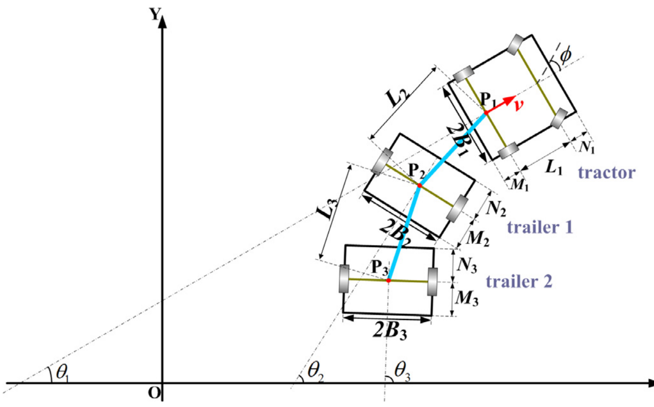

Fig. 2. Schematic of a tractor with $(n-1)$ trailers $(n=3)$. Note that trailer 1 is hooked up at the middle point of the rear wheels of the tractor and trailer 2 is hooked up at that of trailer 1.

orientation angles of every two adjacent parts of the vehicle should be small [2,19–21]. However, jackknifing still occurs when the small-angle constraint is satisfied, as depicted in Fig. 1(b). David and Manivannan [3] summarized that sharp turning, backward movement and fast speed are the three inducing factors that contribute to jackknifing. However, this conclusion is merely based on experiences: jackknifing may occur during forward movements (see Fig. 1(c)).

In summary, the aforementioned studies commonly introduce four drawbacks: (i) time-dependent constraints are not directly handled in the path-planning-first methods; (ii) irregularly placed obstacles in the environment are not precisely described; (iii) only feasible rather than optimized/optimal solutions are generated; (iv) challenges (e.g., reverse instability and jackknifing avoidance) that can be tackled in the open-loop trajectory planning phase are partly transferred to the closed-loop control phase.

In the present study, the original trajectory planning scheme is formulated as a dynamic optimization problem with precisely described kinematics and constraints. Our formulation contains only objective principles rather than human experiences (such as fuzzy logic) [22]. In solving the formulated dynamic optimization problem, we propose a numerical solver that consists of a precise discretization model and a global optimizer. The obtained solutions are strictly feasible.

The remainder of this paper is structured as follows. Section 2 concerns the formulation of a dynamic optimization problem. Then our proposed dynamic optimization solver is introduced in Section 3, followed by Section 4, where simulation results and discussions are presented. Finally, conclusions are drawn in Section 5.

2. Problem formulation

This section focuses on strictly describing the original trajectory generation problem as a dynamic optimization problem, which contains the kinematic principles, mechanical restrictions, collision-avoidance constraints and an optimization objective.

2.1. Kinematics of a tractor-trailer vehicle

This work considers an articulated wheeled vehicle as one car-like front-steering tractor towing $(n-1)$ trailers [23]. Based on the no side slip fundamental assumption, the kinematics of the concerned tractor is described by

$$\begin{cases} \dfrac{dx_1(t)}{dt} = v(t)\cdot\cos\theta_1(t) \\ \dfrac{dy_1(t)}{dt} = v(t)\cdot\sin\theta_1(t)\ , t\in[0,t_f], \\ \dfrac{d\theta_1(t)}{dt} = \dfrac{v(t)\cdot\tan\phi(t)}{L_1} \end{cases} \tag{1}$$

where t refers to time, t_f indicates the unknown terminal moment of the entire movement, (x_1, y_1) denotes the mid-point of rear wheel axis (point P_1 in Fig. 2), θ_1 refers to the orientation angle, v refers to the linear velocity of point P_1 and ϕ refers to the steering angle of front wheels. Moreover, as illustrated in Fig. 2, L_1 denotes the wheelbase length, N_1 denotes the front overhang length, M_1 denotes the rear overhang length and $2B_1$ denotes the tractor width.

Following this, the kinematics of the trailers can be presented as

$$\begin{cases} x_n = x_1 - \sum\limits_{j=2}^{n}\left(L_j\cdot\cos(\theta_j)\right), (n\ge 2) \\ y_n = y_1 - \sum\limits_{j=2}^{n}\left(L_j\cdot\sin(\theta_j)\right), (n\ge 2) \\ \dfrac{d\theta_2(t)}{dt} = \dfrac{v(t)\cdot\sin(\theta_1(t)-\theta_2(t))}{L_2} \\ \dfrac{d\theta_n(t)}{dt} = \dfrac{v(t)\cdot\sin(\theta_{n-1}(t)-\theta_n(t))}{L_n}\cdot \\ \qquad\qquad \left(\prod\limits_{i=1}^{n-2}\cos(\theta_i(t)-\theta_{i+1}(t))\right), (n\ge 3) \end{cases} \quad t\in[0,t_f], \tag{2}$$

where θ_i denotes the orientation angle of the $(i-1)$th trailer, (x_i, y_i) locates the corresponding hooking point P_i, $2B_i$ denotes width of the $(i-1)$th trailer, N_i denotes the corresponding front overhang length, M_i denotes the rear overhang length and L_i denotes the Euclidean distance between two adjacent hook points $(i\ge 2)$.

In the preceding equations, when $v(t)$ and $\phi(t)$ (as well as t_f) are specified, the remaining variables (i.e., $x_i(t)$, $y_i(t)$, and $\theta_i(t)$) can be determined one after another through integral. Therefore, $v(t)$ and $\phi(t)$ are chosen as control variables and the remains are regarded as state variables.

2.2. Mechanical constraints

This subsection focuses on the constraints that should be imposed during the entire maneuver process.

First, we have

$$\begin{cases} \left|v(t)\right| \le \mathrm{v}_{\max}, \\ \left|\phi(t)\right| \le \Phi_{\max}, \end{cases} \quad t\in[0,t_f]. \tag{3}$$

The reasons for imposing boundaries on $v(t)$ and $\phi(t)$ are evident: (i) the linear velocity of the vehicle is expected to be not too fast and (ii) the steering angle of the tractor is mechanically limited. Second, those two control variables should be continuous. Third, for such a multi-body vehicle, different parts should not collide with each other. Assuming that the tractor and trailers are rectangular, how to strictly formulate this collision-free condition is introduced next.

Basically, let us investigate the judgment whether two rectangles collide. All of the possibilities that one rectangle collides with another