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Model reduction for combined surface water/groundwater management formulations

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A B S T R A C T

A methodology for clustering stresses on a groundwater system based on similarity of hydrologic response produced by the stress is presented. The method is relevant to the computationally efficient calculation of response matrixes for use in groundwater management and other applications. The procedure is presented for the case in which the impact of pumping withdrawals on streamflow is of interest. The method uses cluster analysis on multiple, transient responses and a simplified response matrix for the modeled system. It is demonstrated on a field scale hypothetical management problem applied to a portion of the Republican River basin in the High Plains Aquifer of the United States with a multi-decadal planning horizon that maximizes well withdrawal while requiring that minimum streamflows be maintained. For this case, the clustering approach both reduces the computational requirement and helps to produce response coefficients with meaningful precision. The effectiveness of the model reduction is tested by comparing the optimal solutions produced by a full scale formulation and two reduced size formulations.

1. Introduction

Determining the response of the state of a groundwater system to imposed stresses is a fundamental problem in groundwater hydrology. A common approach is to use a calibrated simulation model to estimate the system state both before and after the application of the stress. In this paper, we focus on the impact of withdrawals from pumping wells on streamflow in nearby streams. This particular pair of stress and system state is of wide interest in irrigated agriculture and municipal water supply applications and a frequent topic for modeling studies (e.g. Bredehoeft, 2011; Arnold et al., 1993; Sophocleous et al., 1999; Rossman and Zlotnik, 2013; Maxwell et al., 2014). In these cases, large scale pumping can have impacts on streamflow years and even decades after pumping has ceased (Barlow and Leake, 2012).

A useful approach to response determination is the creation of response functions and response maps. A response function describes the response of a given system state to withdrawals as a function of time. For withdrawal and streamflow, this relationship is referred to as the streamflow depletion curve (Barlow and Leake, 2012). A response map provides a geographic depiction of the relationship between hypothetical withdrawal at any location in an aquifer domain on streamflow at a specific point. Leake et al. (2010) describe a method for determining a response map specific to streamflow and pumping referred to as a capture map. Typically, each numerical cell in the grid is considered as a possible location for a stress.

Another use of response information is in the formulation of groundwater management models, also known as simulation/optimization models, (Gorelick et al., 1993; Ahlfeld and Mulligan, 2000). Management models identify stresses as decision variables whose value is determined by an optimization algorithm. The algorithm optimizes an objective function subject to constraints on decision variables and system state. A common approach to solving management models is the response matrix method (Chan, 1993; Ahlfeld et al., 2005; Yeh, 2015). This approach uses the response matrix as a surrogate model of the full simulation model. The resulting optimization problem can be solved using standard methods such as linear programming (Dantzig, 1963) or successive linear programming (Stewart and Griffith, 1961; Ahlfeld and Baro-Montes, 2008). Applications which manage well withdrawals to control streamflow depletion are numerous (Maddock, 1974; Morel-Seytoux and Daly, 1975; Illangasekare and Morel-Seytoux, 1982; Peralta et al., 1988; Mueller and Male, 1993; Barlow et al.,

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2003; Cosgrove and Johnson, 2005; Elbakidze et al., 2012; Gorelick and Zheng, 2015).

A response map can be organized as a response matrix by associating a column of the response matrix with each cell that produces a response. Each row of the response matrix is associated with a system state. For a single system state, such as streamflow at a particular location and time, the response matrix would contain a single row and one column for each of the cells in the grid. If system state over multiple time periods at a single location is to be determined (as in a response function) then the response matrix would have a row for each time period and a column for each cell. For such a response matrix, the columns contain the information displayed in a response functions and the rows contain the information displayed in a response map.

Response matrices used in groundwater management models generalize the concepts of response maps and functions further by using multiple system state locations over multiple time periods. Whereas a response map displays the response of a single system state to steady pumping or pumping over a limited time period, in large-scale management problems, it may be necessary to understand the response of system state to pumping at each pumping location and in each of many time periods (e.g. annual irrigation withdrawals). Such is the case in the case study presented below.

Creation of response maps, functions or matrices for simulation models with large spatial domains and long time horizons can be computationally intensive. As will be shown for the application presented herein, the full management model yields a problem requiring thousands of forward runs of the simulation model for creation of the response matrix. These computational demands motivate investigation of methods for model reduction; the formulation of a problem with reduced size that is a good approximation to the original problem.

The approach to model reduction taken in this paper is to cluster withdrawals into groups that have similar hydrologic impact. Withdrawals clustered into a single group can then be treated as a single stress requiring a single forward run of the simulation model for determination of impact. In the context of the management model this means that multiple decision variables will be clustered to form a single decision variable that represents withdrawal over multiple cells. The K-means clustering method (Hartigan, 1975) will be used to group withdrawals. Cosgrove and Johnson (2005) used a similar method to aggregate groundwater users into zones of similar stream impacts. The authors used steady-state stream depletion response applied to the eastern Snake River Plain in southern Idaho as the basis for the clustering.

In the present paper, the management model is formulated for an irrigation management problem in the High Plains Aquifer in central North America. The groundwater and surface water system is modeled using the 3-D finite difference groundwater flow model MODFLOW (Harbaugh, 2005). A method is described to compute a preliminary response matrix that is used for the clustering analysis. After clustering, the Reduced Model is compared to a full-scale Model which was run for the purpose of this paper.

In this paper we examine two levels of model reduction to determine if reasonable results can be produced using substantially fewer decision variables. We speculate that reducing the number of decision variables to 100 will produce closely comparable results. A second clustering down to 50 decision variables is used to confirm that smaller values tend to degrade the results.

In section 2, an overview of response matrices and their computation is presented. In section 3, the hypothetical model which provides the test case for the reduced model analysis is presented. Section 4, describes the clustering methodology and the specific results for the case study. Section 5 compares the results of the optimization for the full formulation and the two reduced formulations.

1.1. The response matrix

Each element of a response matrix is referred to as a response coefficient. The value of the response coefficient indicates the change in the value of the system state to a unit change in a stress. For the present study, the system state will be streamflow, S, and the stress will be withdrawal, Q. The response coefficient is most commonly computed using the simulated change in streamflow resulting from a unit change in withdrawal. Eq. (1) describes the calculation of the response coefficient using the perturbation method (Ahlfeld and Mulligan, 2000) also called the algebraic technological function (Maddock, 1972):

$$r_{i,j} = \frac{S_i\left(\boldsymbol{Q}_{\Delta j}\right) - S_i^0\left(\boldsymbol{Q}^0\right)}{Q_{\Delta j}} \tag{1}$$

An initial or base run of the simulation model computes base streamflow values, S_i^0, at each location i, using the base withdrawal values $\boldsymbol{Q}^0$. The index i maps to both a spatial and temporal location. Changing or perturbing the withdrawal rate for only one of the stresses, the model run is repeated. Here, $Q_{\Delta j}$ is the change in the withdrawal rate for withdrawal location j. The index j maps to both a spatial location and a time period over which withdrawal is active. $S_i(\boldsymbol{Q}_{\Delta \boldsymbol{j}})$ is the streamflow at location i evaluated with the set of withdrawal rates in which all rates are at their base value except for the jth withdrawal which has been perturbed by the amount $Q_{\Delta j}$. The difference in computed streamflow at each location is divided by the change in withdrawal that produced the change to yield the response coefficient, $r_{\boldsymbol{i,j}}$, for the impact of withdrawal at location j on streamflow at location i.

The response coefficients can be assembled into a response matrix. Each column of the response matrix is associated with a withdrawal point, j. Each row of the response matrix is associated with a streamflow location, i. This organization of the response coefficients makes it easy to recognize two important characteristics of the response matrix.

First, computation of the response matrix is driven by the number of columns in the matrix; which is equal to the number of different withdrawal spatiotemporal locations. Using the perturbation method, and assuming that the base run has been conducted, it is sufficient to perform a single run of the simulation model, using the perturbed withdrawal rate and observe, from the model output, the streamflow at all spatiotemporal locations of interest to populate the entire column of the response matrix.

Second, the rows of the response matrix contain all the information required to produce a predictive model for streamflow at location i based on the principle of superposition. Specifically, for a given set of withdrawal rates, the streamflow at location i can be estimated according to:

$$S_i = S_i^0 - \sum_j \left[r_{i,j} Q_j\right] \tag{2}$$

Where the jth term in the summation represents the change in streamflow produced by pumping at the rate Q_j. When (2) can be considered a reasonable approximation, it forms the core of the response matrix method. Streamflows that appear in the objective function or constraints can be replaced with the right side of (2) and written as a linear function of withdrawal rate decision variables. Equation (2) may not be a reasonable approximation if significant nonlinear responses are present through such features as boundary conditions or transmissivities that are head dependent. When nonlinear features are present the value of the perturbation